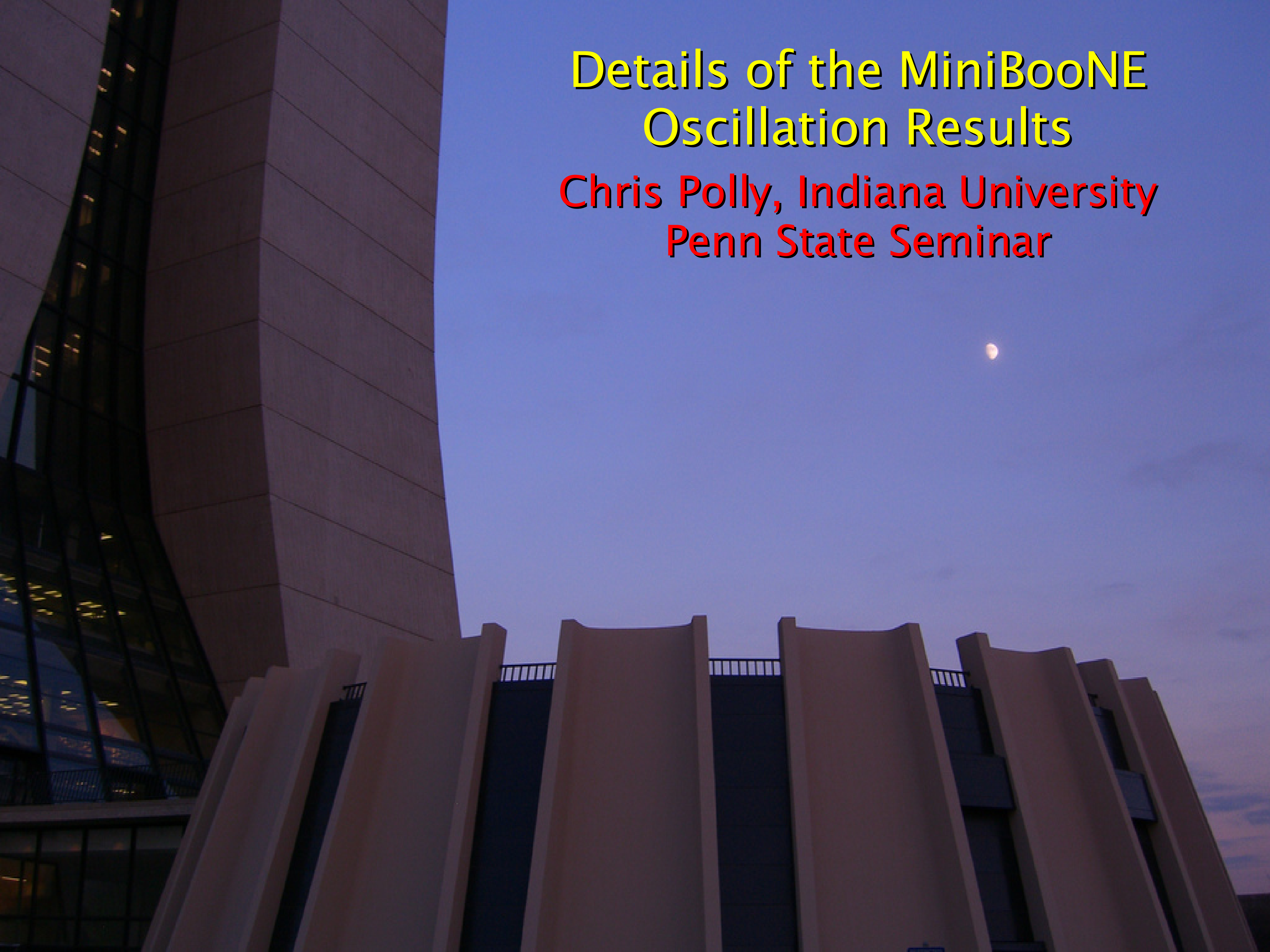
Details of the MiniBooNE Oscillation Results

Chris Polly, Indiana University

Penn State Seminar



The MiniBooNE Collaboration

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Los Alamos National Laboratory
Louisiana State University
University of Michigan
Princeton University
Saint Mary’s University of Minnesota
Virginia Polytechnic Institute
Western Illinois University
Yale University

★ Thanks to Doug Cowen for the invitation...

Neutrino Oscillations

- ν oscillations first postulated by Pontecorvo in 1957, based on analogy to kaons.
- A non-zero ν mass allows for lepton flavor changes.
- mass eigenstates ≠ flavor eigenstates:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \qquad \alpha = (e, \mu, \tau)$$

- Flavor composition changes as ν propagates:

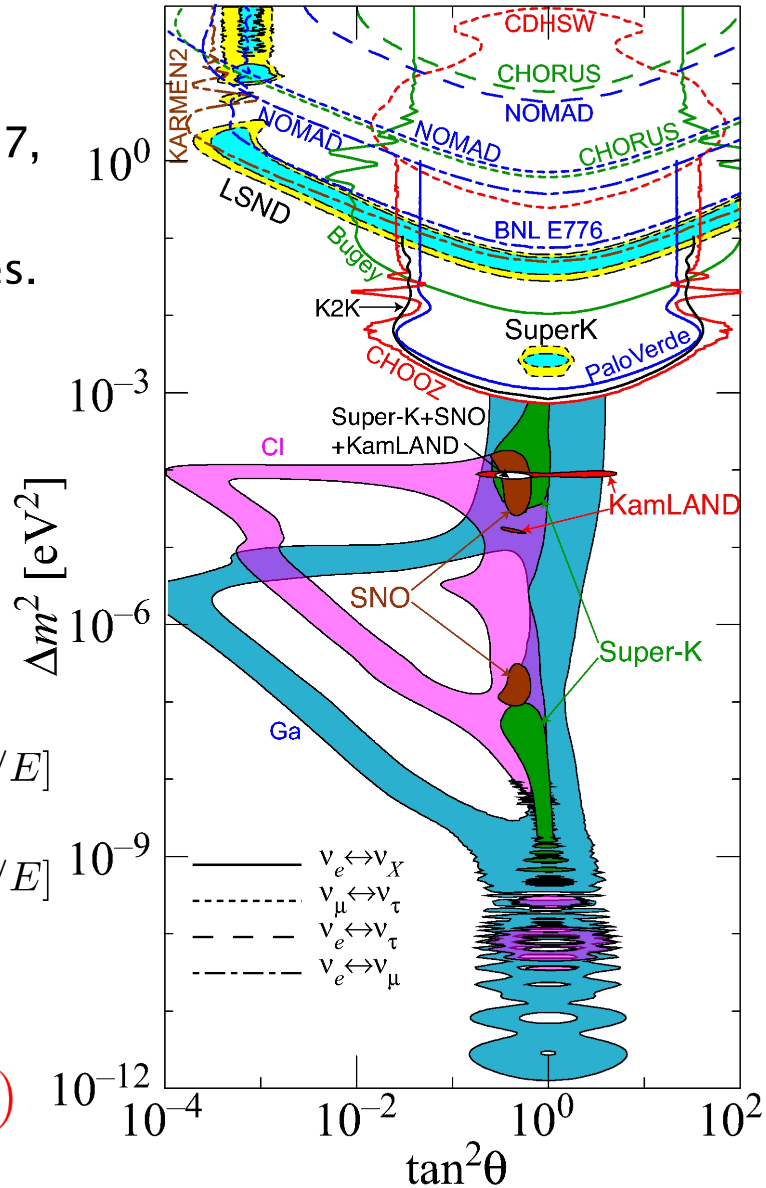
$$P(\nu_\alpha \to \nu_\beta) = |\langle\nu_\beta|\nu_\alpha(L)\rangle|^2$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2[1.27\Delta m^2_{ij}L/E]$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2[2.54\Delta m^2_{ij}L/E]$$

- Reducing to simple 2-neutrino mixing:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2(1.27\Delta m^2 L/E)$$

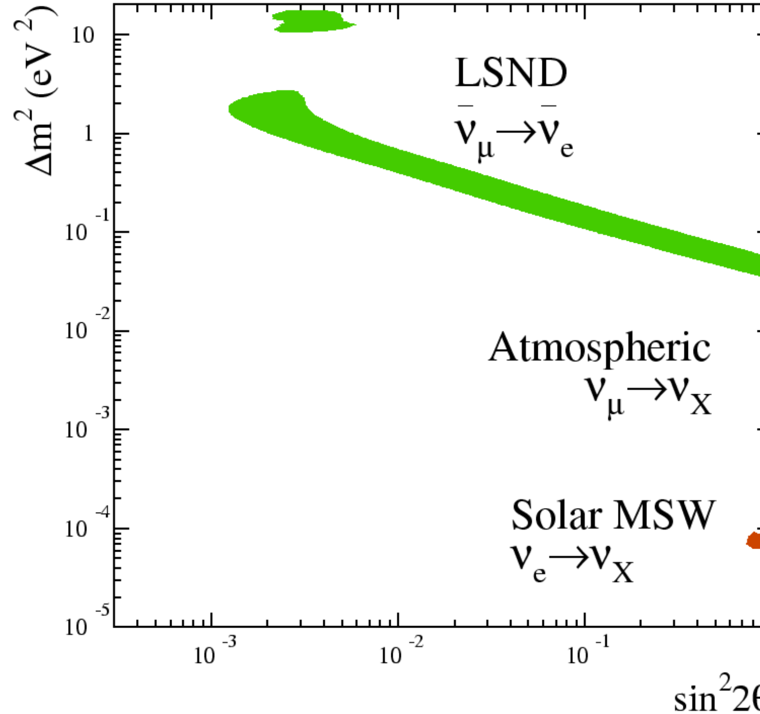


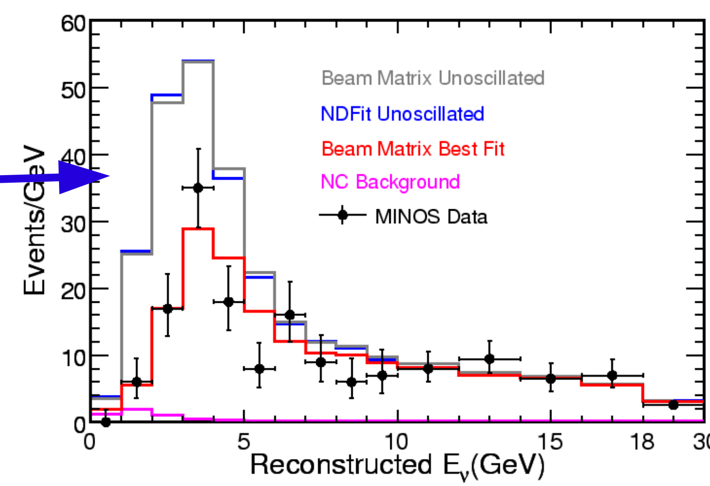
http://hitoshi.berkeley.edu/neutrino

➔ Many experiments have hunted for ν oscillations, some have found them!

Evidence for ν oscillations

- First evidence came in 1968 from Davis' solar ν_e experiment
  - found 1/3 of the expected ν_e from sun
  - disappearance $\nu_e \to \nu_X$
  - $\Delta m_{12}^2 \sim 8\times10^{-5}$ eV^2, $\sin^2(2\theta) \sim 0.8$
  - Confirmed by SNO, Super-K, Kamland
- New mixing found by Super-K through atmospheric ν_μ oscillations
  - found 1/2 as the upward ν_μ as downward
  - disappearance $\nu_\mu \to \nu_X$
  - $\Delta m_{23}^2 \sim 2\times10^{-3}$ eV^2, $\sin^2(2\theta) \sim 1.0$
  - Confirmed by IMB, Soudan, K2K, and most recently MINOS
- Only one unconfirmed observation!

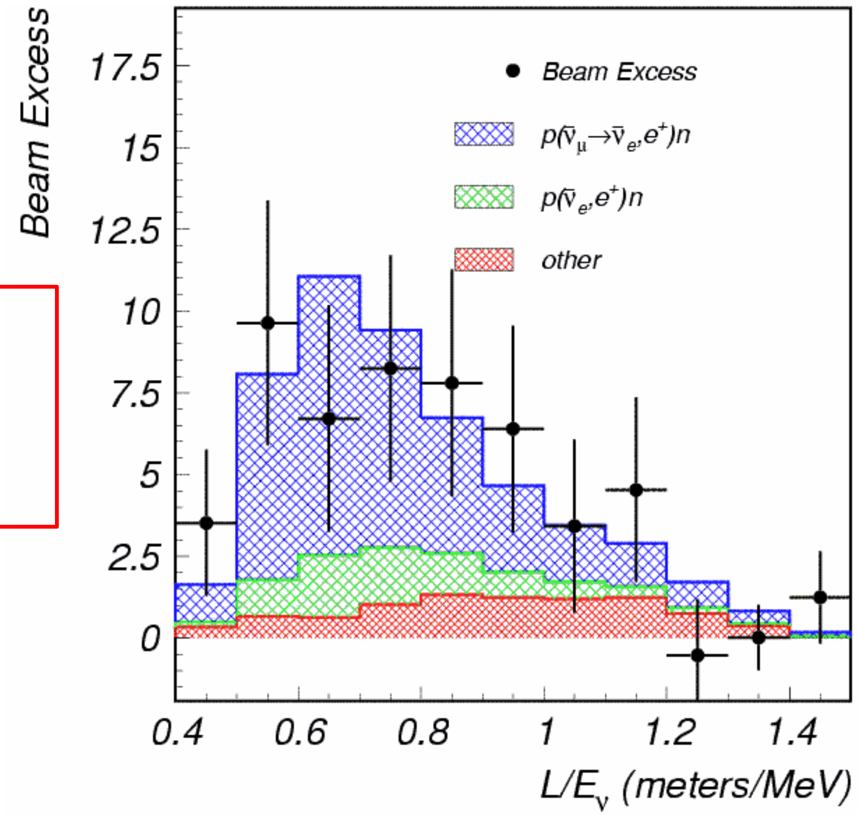




MiniBooNE's motivation...LSND

- LSND found an excess of $\overline{\nu}_e$ in $\overline{\nu}_\mu$ beam
- Signature: Cerenkov light from e^+ with delayed n-capture (2.2 MeV)
- Excess: 87.9 ± 22.4 ± 6.0 (3.8σ)
- Under a 2ν mixing hypothesis:

$$P(\overline{\nu}_\mu \to \overline{\nu}_e) = \sin^2(2\theta)\sin^2\left(\frac{1.27\, L\, \Delta m^2}{E}\right)$$
$$= 0.245 \pm 0.067 \pm 0.045\ \%$$



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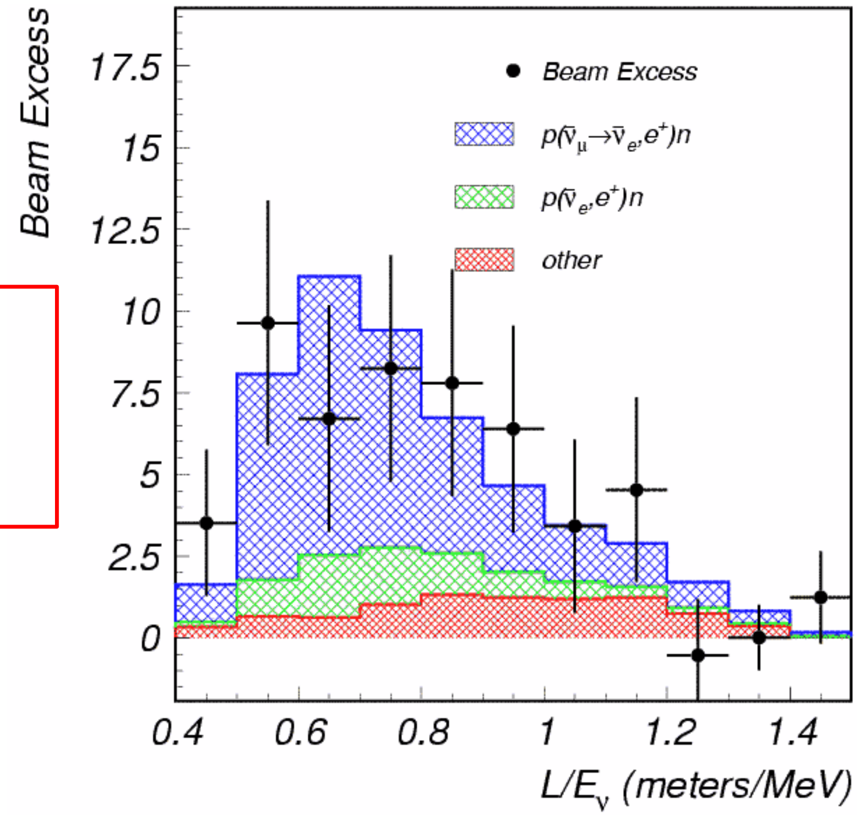
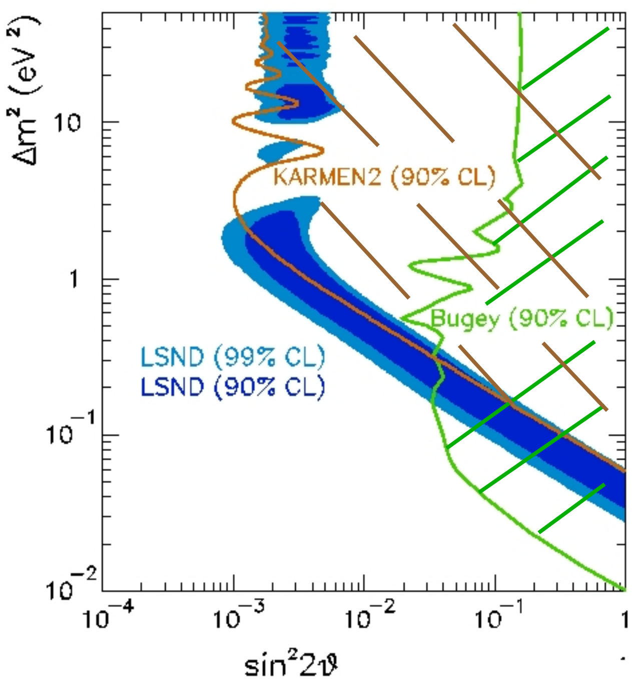
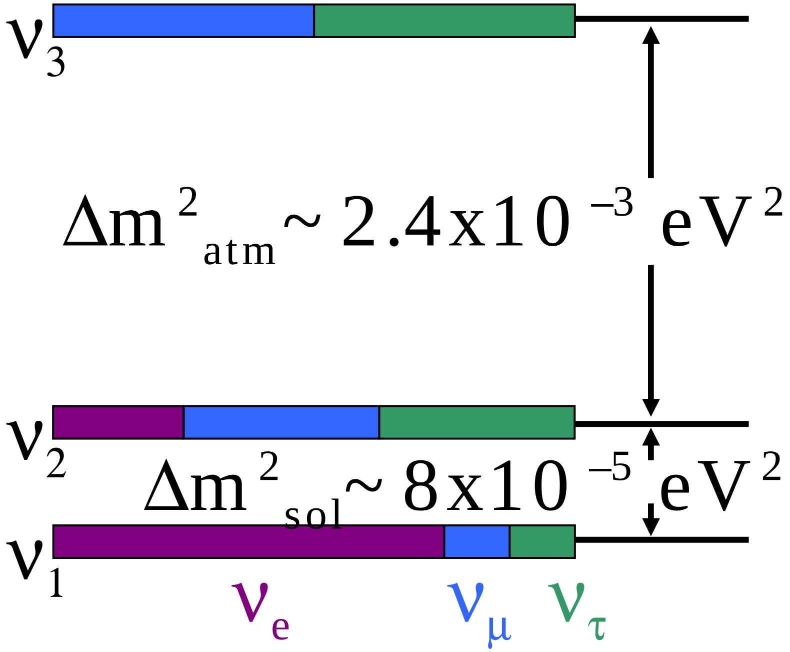




- Other experiments, i.e. Karmen and Bugey, have ruled out portions of the LSND signal
- MiniBooNE was designed to cover the entire LSND allowed region

Interpreting the LSND signal



- The other two measured mixings fit conveniently into a 3-neutrino model
- With $\Delta m_{13}^2 = \Delta m_{12}^2 + \Delta m_{23}^2$, the LSND $\Delta m^2 \sim 1$ eV^2 does not fit
- 'Simplest' explanation...a 4th neutrino

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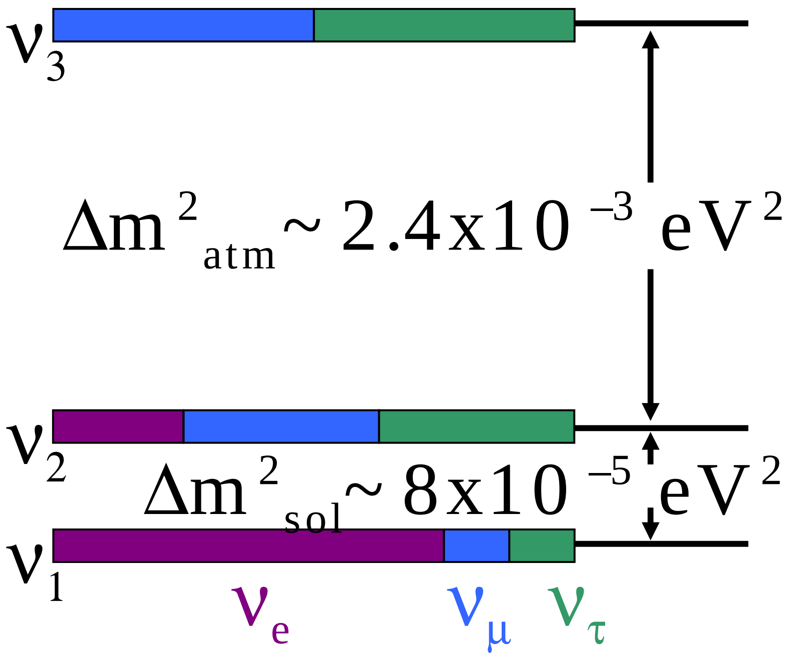

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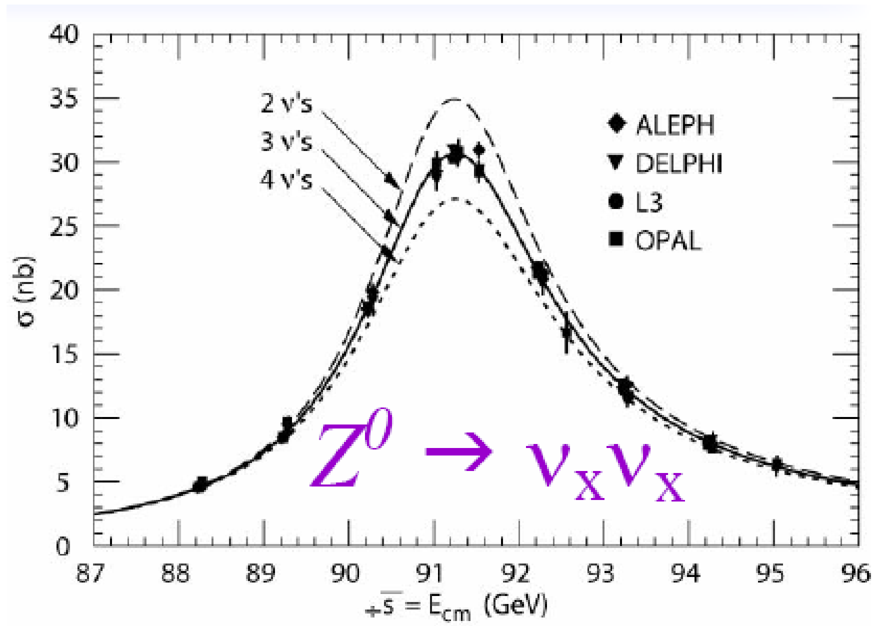
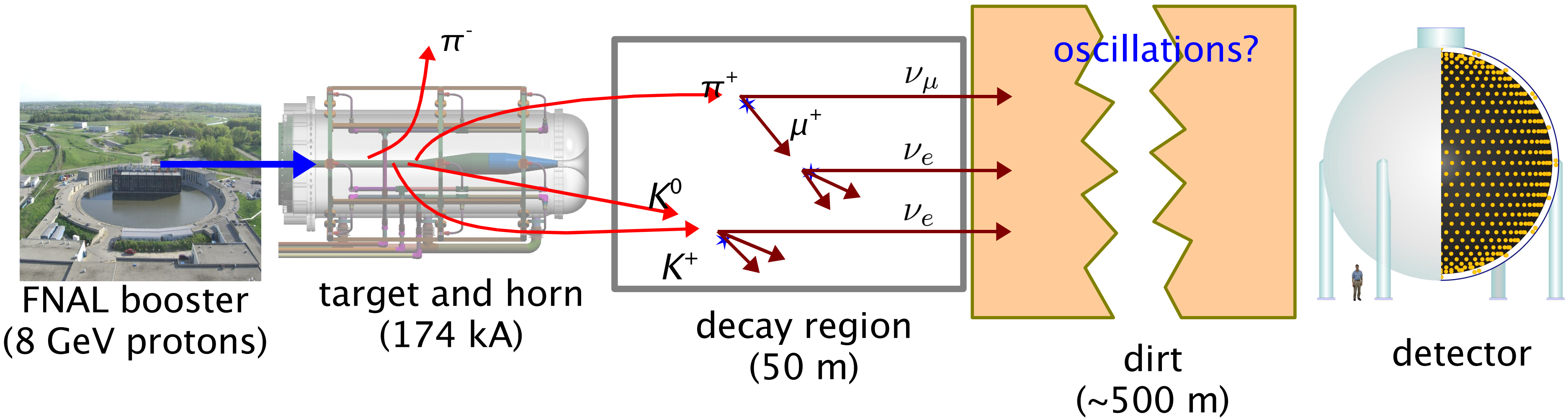
- Width of the Z implies 2.994 + 0.012 light neutrino flavors
- Requires 4th neutrino to be 'sterile' or an even more exotic solution
  - Sterile neutrinos *hep-ph/0305255*
  - Neutrino decay *hep-ph/0602083*
  - Lorentz/CPT violation *hep-ex/0506067*
  - Extra dimensions h*ep-ph/0504096*



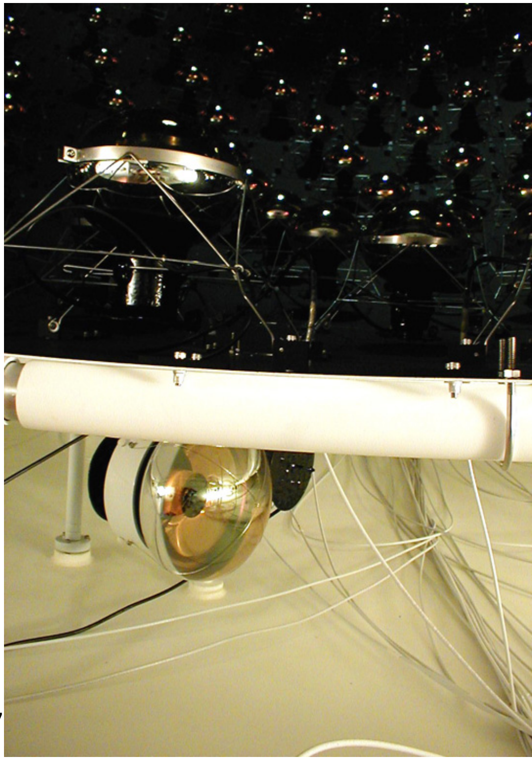
The MiniBooNE design strategy



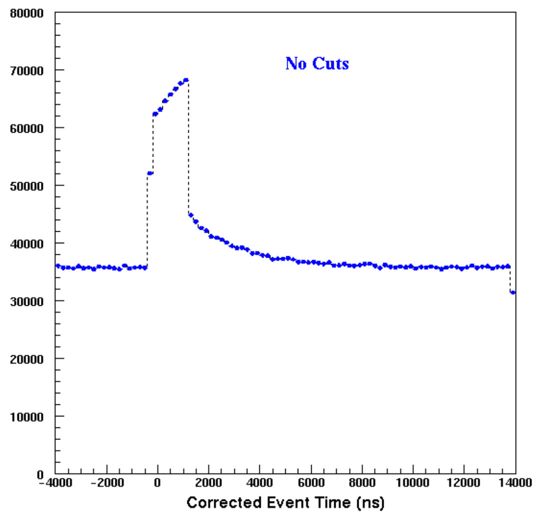
- Start with 8 GeV proton beam from FNAL Booster
- Add a 174 kA pulsed horn to gain a needed x 6
- Requires running ν (not anti-ν) to get flux
- Pions decay to ν with E_ν in the 0.8 GeV range
- Place detector to preserve LSND L/E:

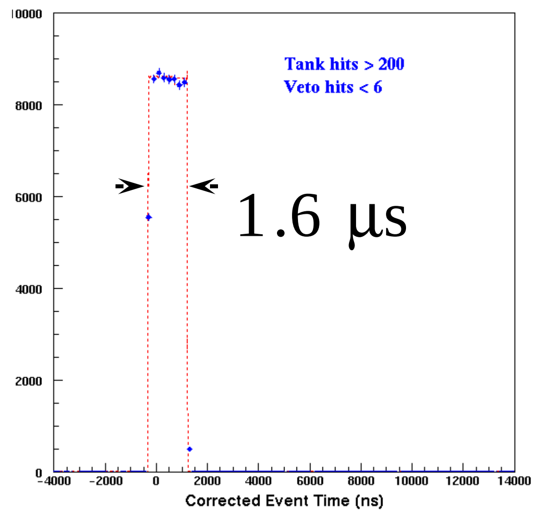
MiniBooNE:	(0.5 km) / (0.8 GeV)
LSND:	(0.03 km) / (0.05 GeV)

- Detect ν interations in 800T pure mineral oil detector
  - 1280 8” PMTs provide 10% coverage of fiducial volume
  - 240 8” PMTs provide active veto in outer radial shell



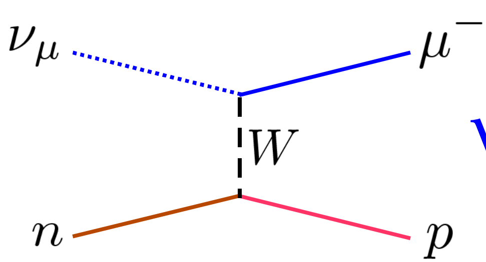
Simple cuts eliminate random backgrounds



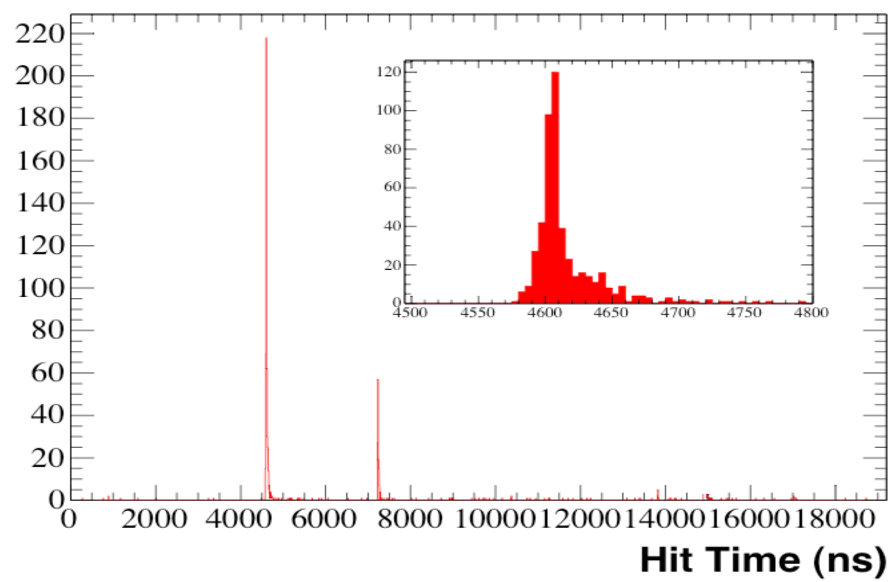


- Left: trigger window, no cuts
- Right: Simple cuts applied PMT hits in veto < 6 and tank > 200 show clean beam window
- Removes cosmic μ and their decay electrons

- Subevent structure (clusters in time) can be used for particle identification (PID)
- Time structure on right expected for most common ν interaction in MB: ν_μ charged-current quasi-elastic

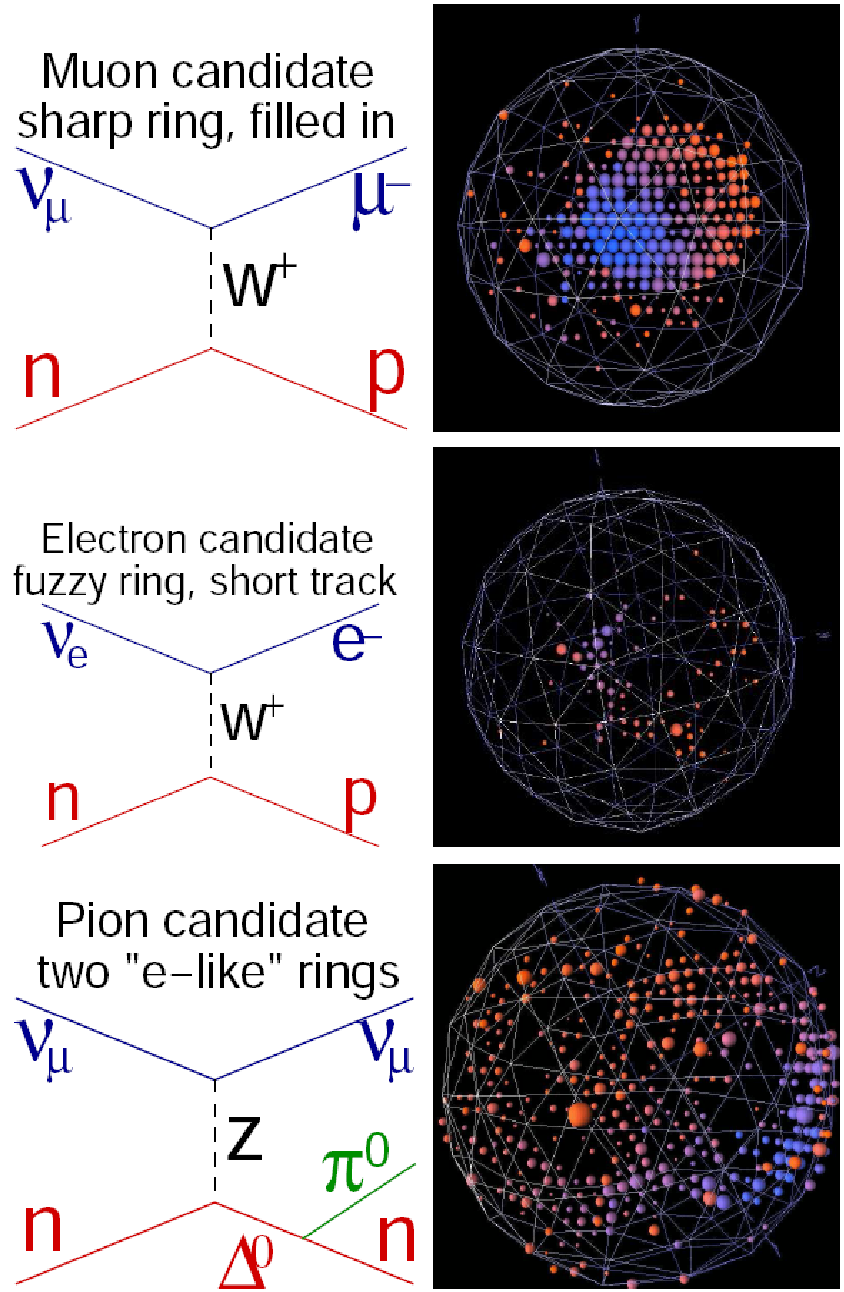


ν_μ CCQE

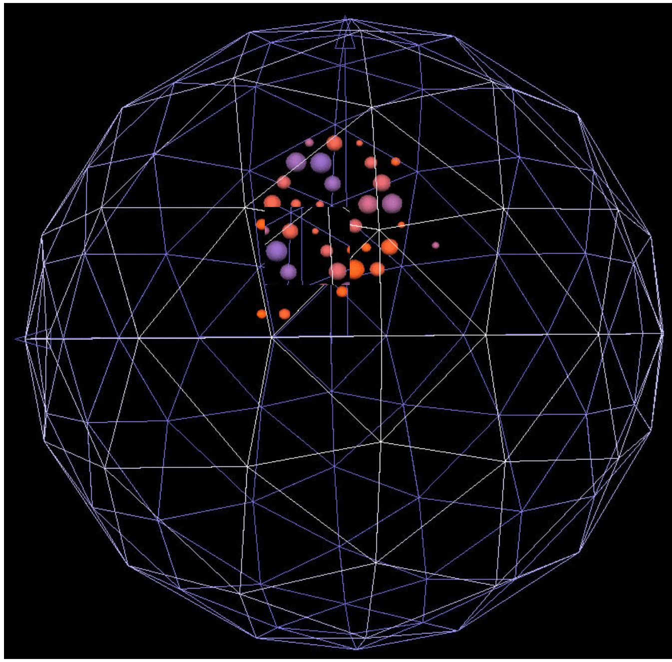


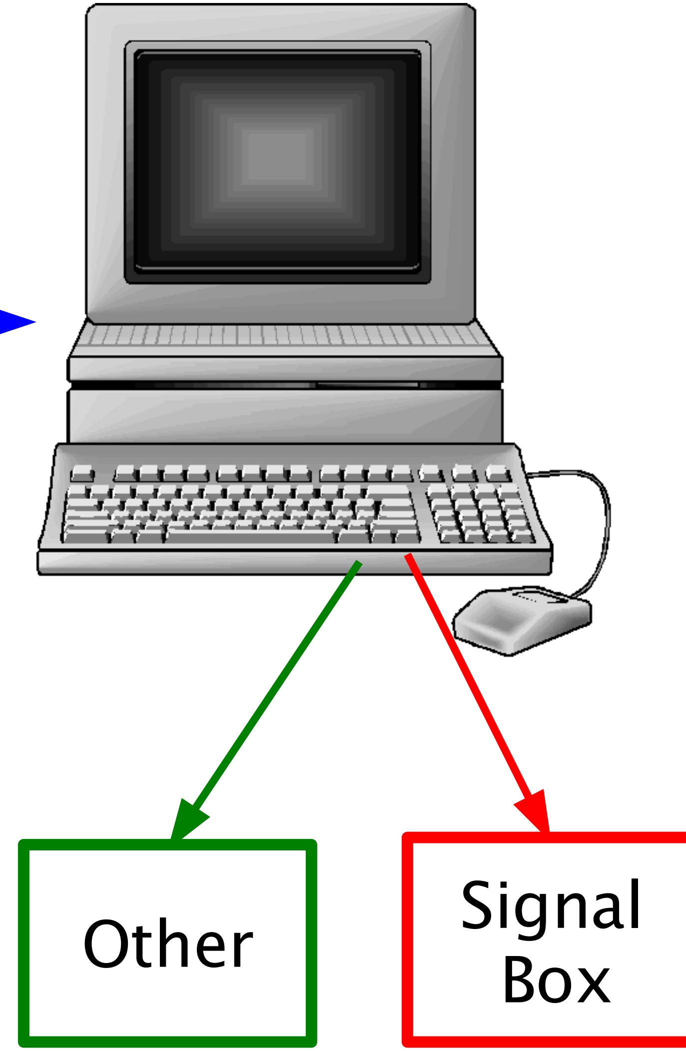
Key points about the signal

- LSND oscillation probability is < 0.3%
- After cuts, MiniBooNE has to be able to find ~300 ν_e CCQE interactions in a sea of ~150,000 ν_μ CCQE
- Intrinsic ν_e background
  - Actual ν_e produced in the beamline from muons and kaons
  - Irreducible at the event level
  - E spectrum differs from signal
- Mis-identified events
  - ν_μ CCQE easy to identify, i.e. 2 “subevents” instead of 1. However, lots of them.
  - Neutral-current (NC) π^0 and radiative Δ are rarer, but harder to separate
  - Can be reduced with better PID
- MiniBooNE is a ratio measurement with the ν_μ constraining flux X cross-section



Blind analysis in MiniBooNE

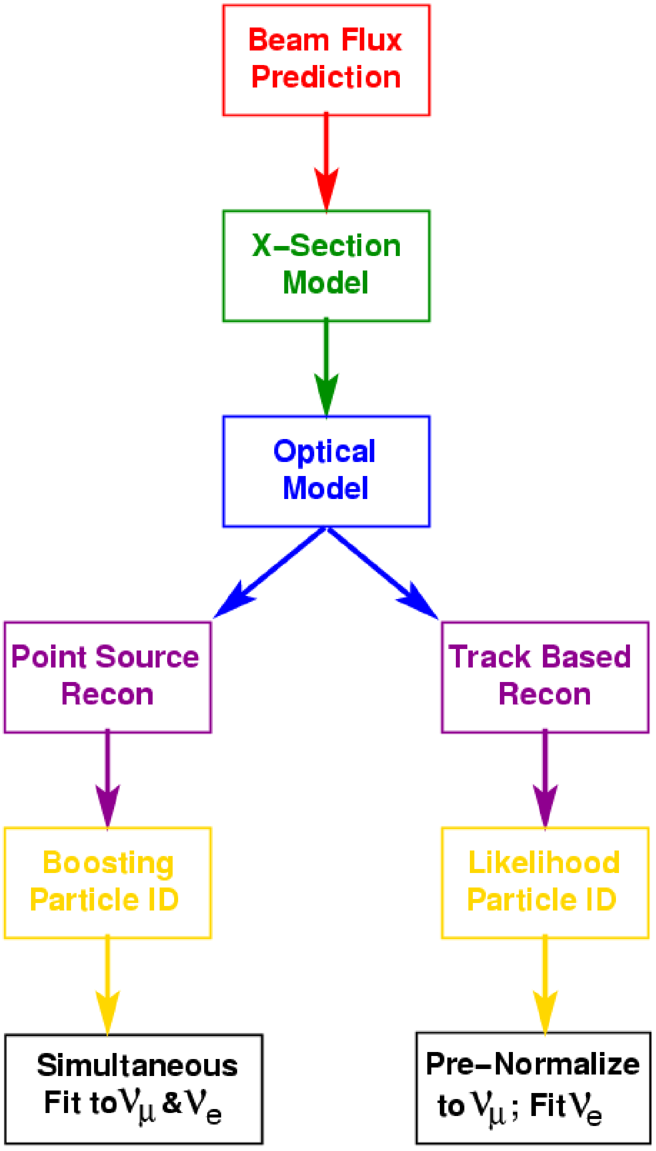




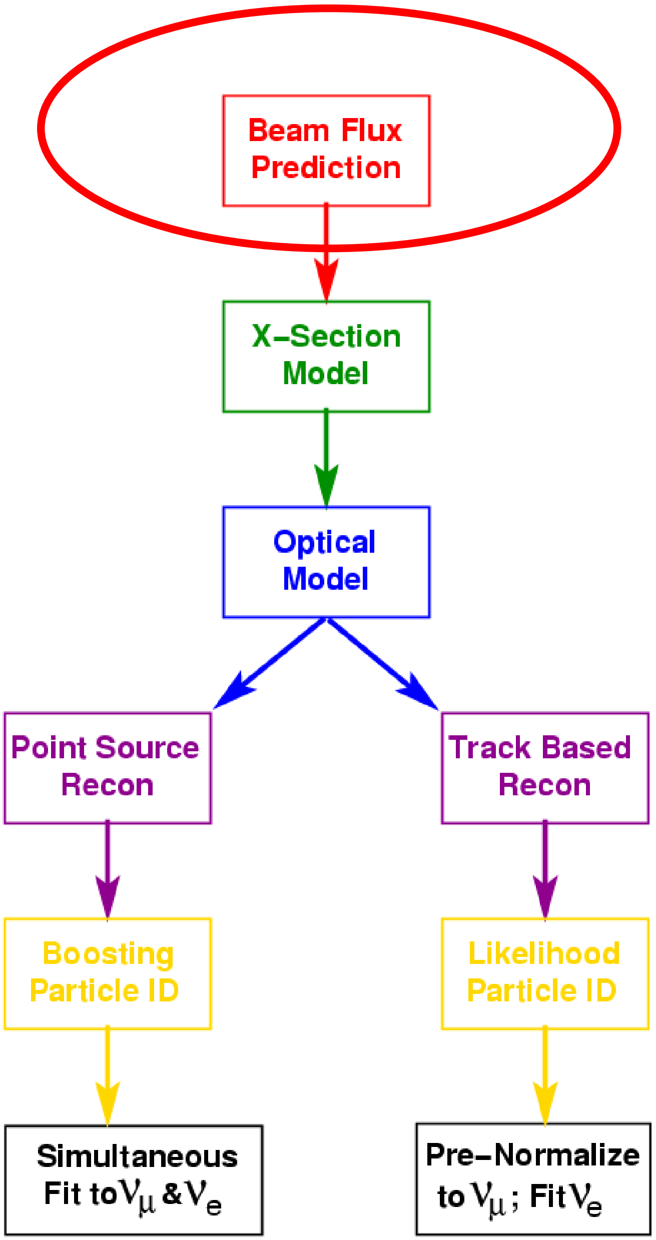
- The MiniBooNE signal is small but relatively easy to isolate
- As data comes in it is classified into 'boxes'
- For boxes to be opened to analysis they must be shown to have a signal < 1σ
- In the end, 99% of the data were available prior to unblinding...necessary to understand errors

MiniBooNE analysis structure

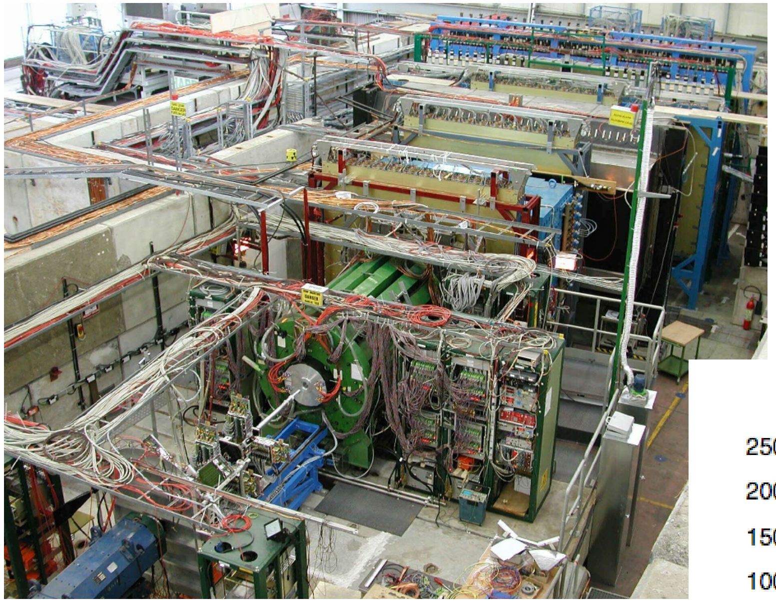
- ✔ Start with a Geant 4 flux prediction for the ν spectrum from π and K produced at the target
- ✔ Predict ν interactions using Nuance
- ✔ Pass final state particles to Geant 3 to model particle and light propagation in the tank
- ✔ Starting with event reconstruction, independent analyses form: Boosted Decision Tree (BDT) and Track Based Likelihood (TBL)
- ✔ Develop particle ID/cuts to separate signal from background
- ✔ Fit reconstructed E_ν spectrum for oscillations



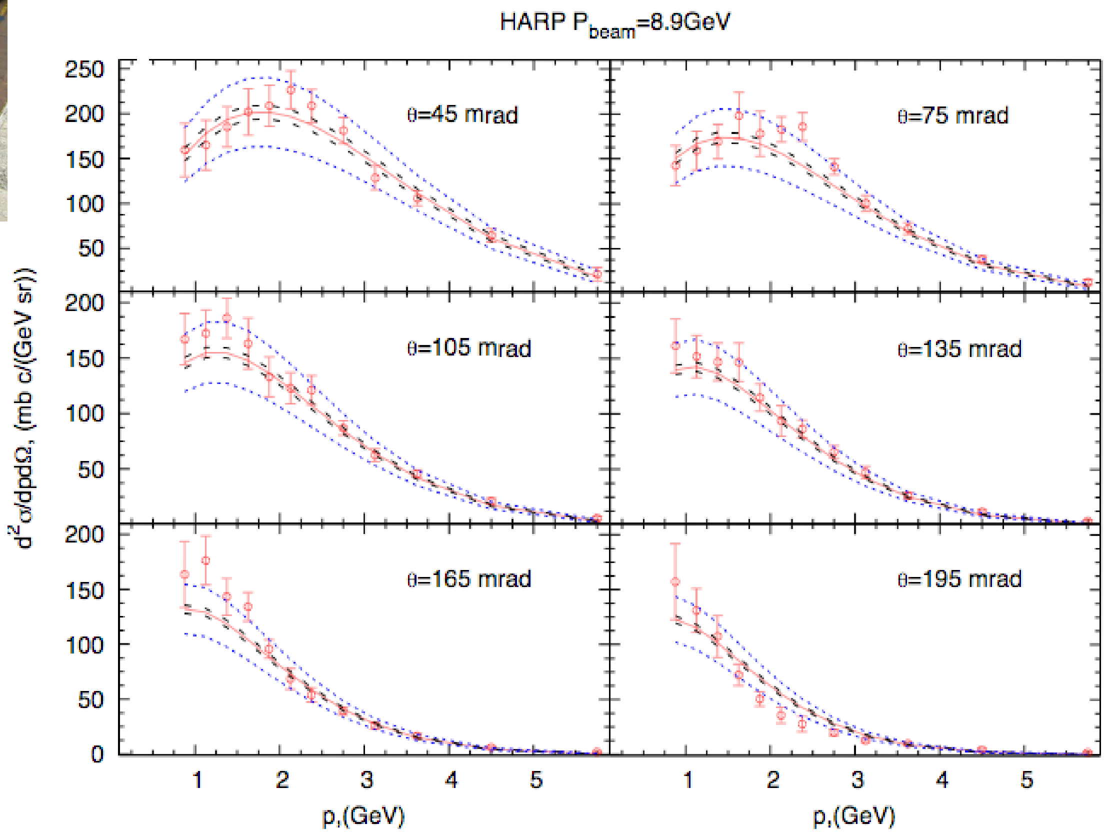
Flux Prediction



Modeling pion production



- HARP (CERN)
  - 5% λ Beryllium target
  - 8.9 GeV proton beam momentum



Data are fit to a Sanford-Wang parameterization.

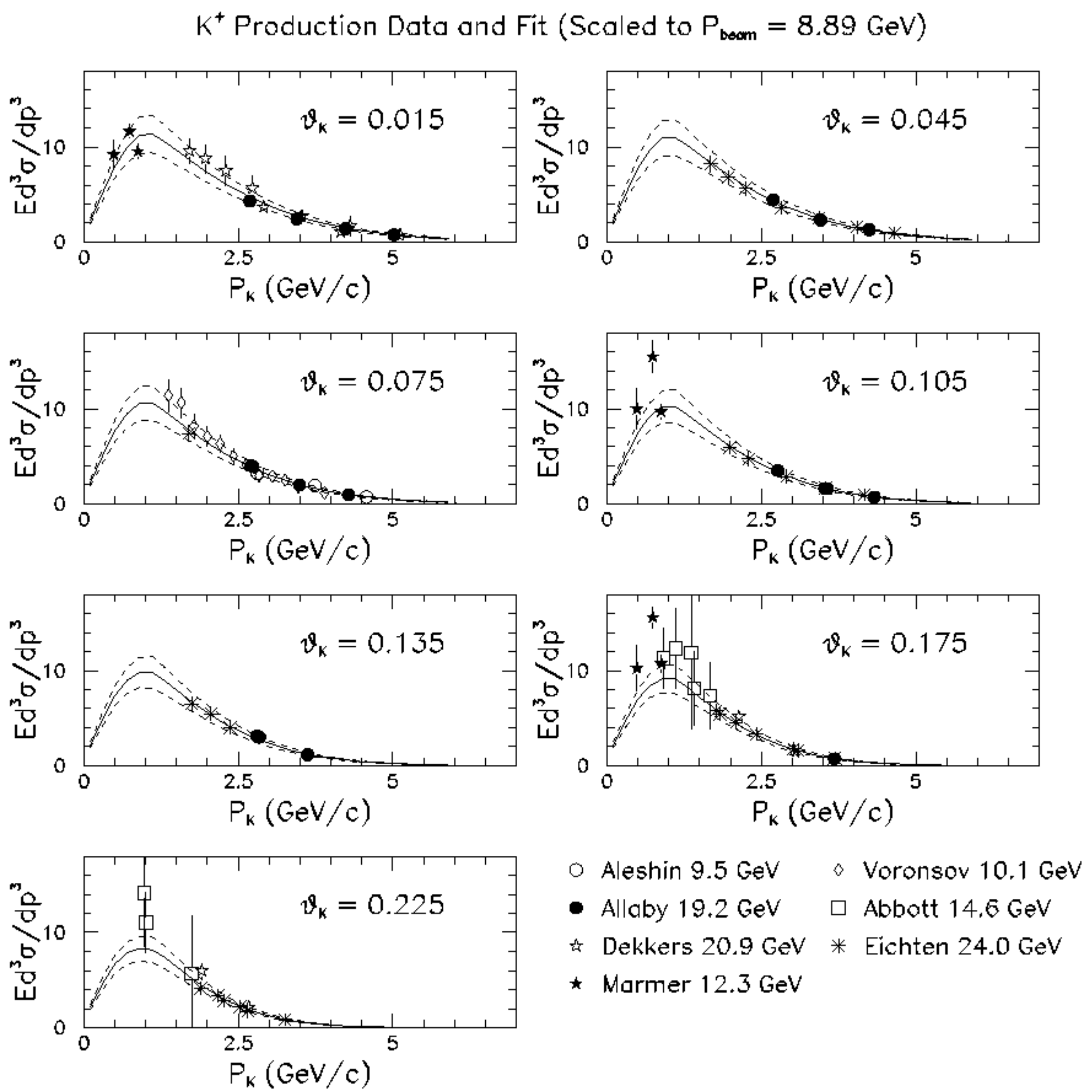
HARP collaboration, hep-ex/0702024

Modeling kaon production

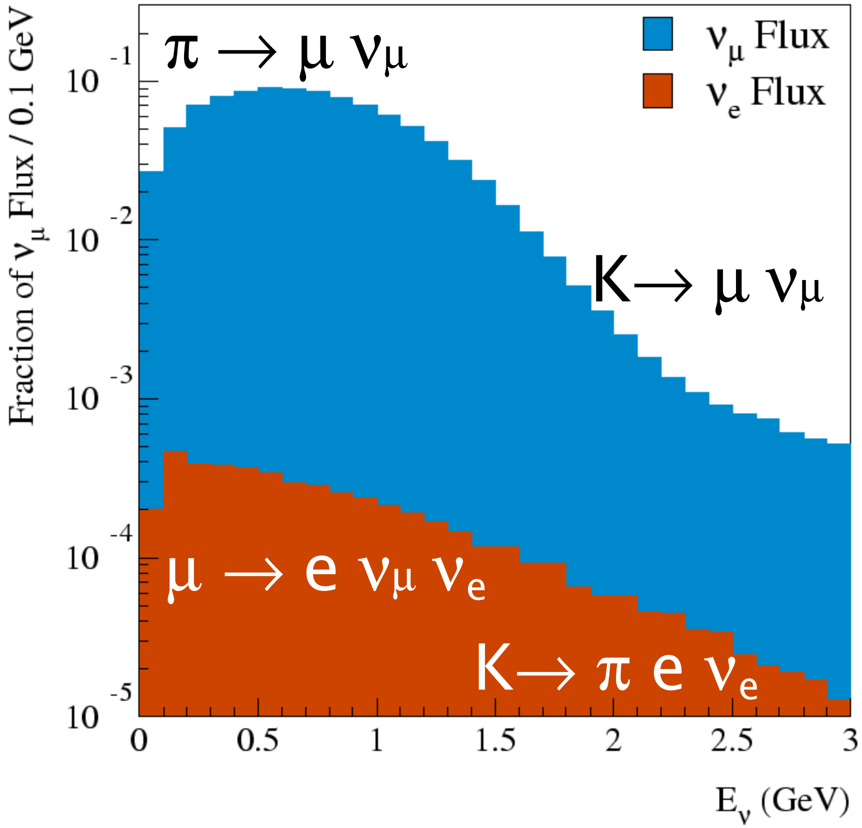
K^+ Data from 10 – 24 GeV uses a Feynman scaling parameterization.

data -- points
dash --total error
(fit ⊕ parameterization)

K^0 data are also parameterized.



Final neutrino flux estimation



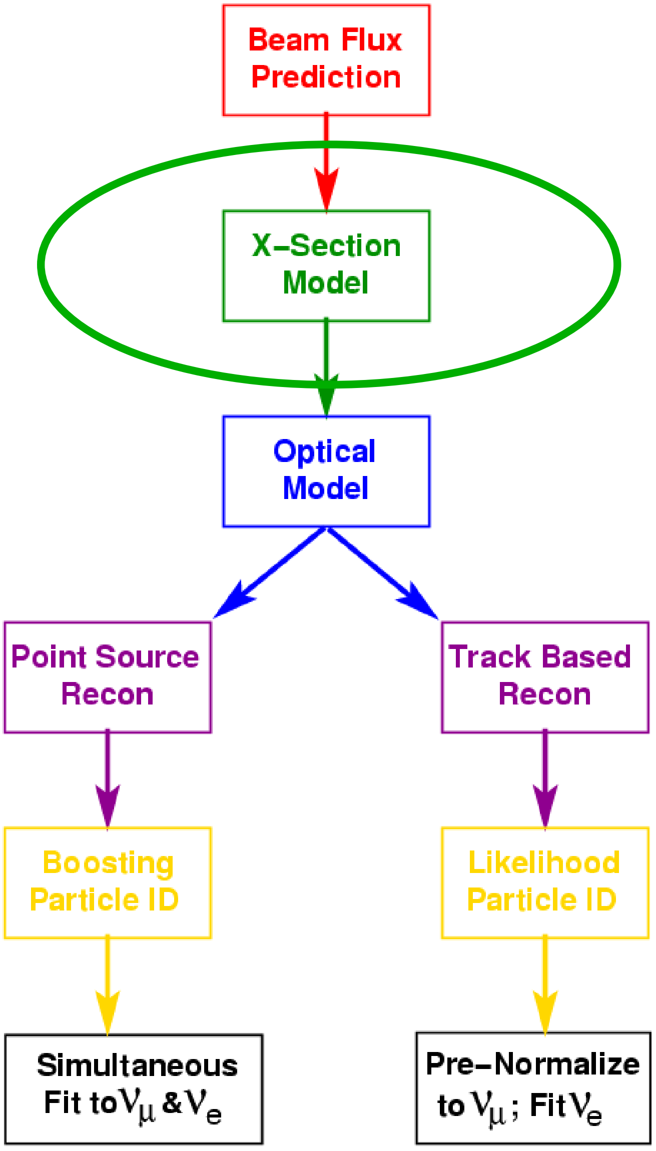
$\nu_e / \nu_\mu = 0.5\%$

"Intrinsic" $\nu_e + \bar{\nu}_e$ sources:

$\mu^+ \rightarrow e^+\ \bar{\nu}_\mu\, \nu_e$	(52%)
$K^+ \rightarrow \pi^0\, e^+\ \nu_e$	(29%)
$K^0 \rightarrow \pi\, e\, \nu_e$	(14%)
Other	(5%)

Antineutrino content: 6%

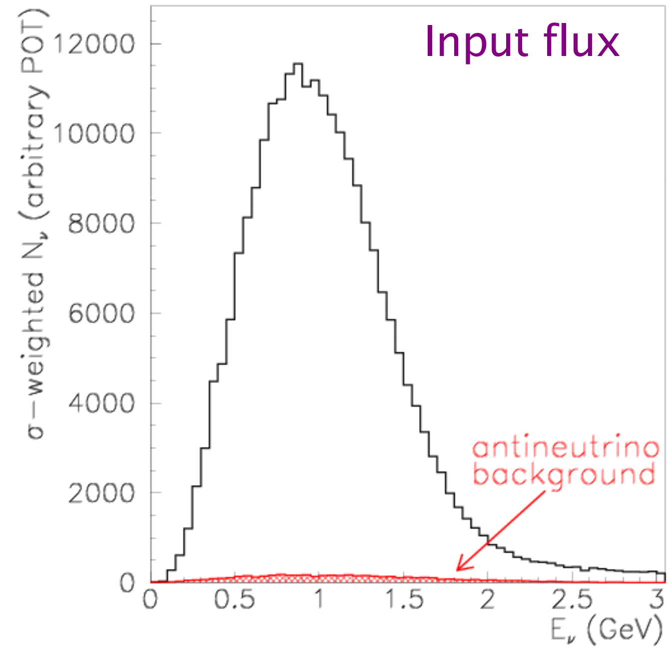
X–Section Model

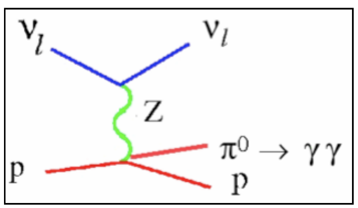


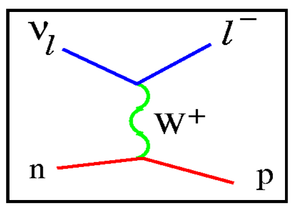
Nuance Monte Carlo

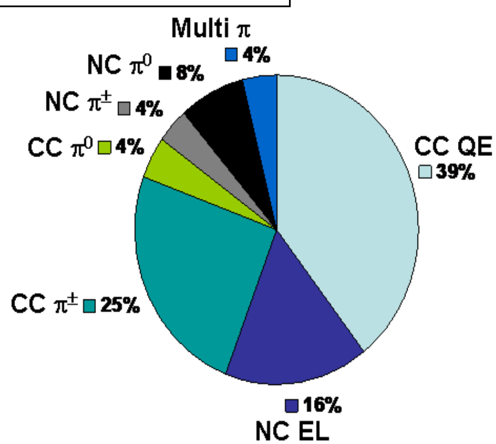
D. Casper, NPS, 112 (2002) 161

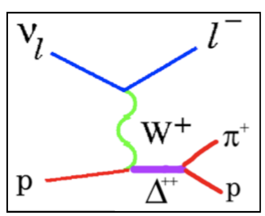
- Comprehensive generator, covers entire E_ν range
- Predicts relative rate of specific ν interactions from input flux
- Expected interaction rates in MiniBooNE (before cuts) shown below
- Based on world data, ν_μ CC shown below right
- Also tuned on internal data

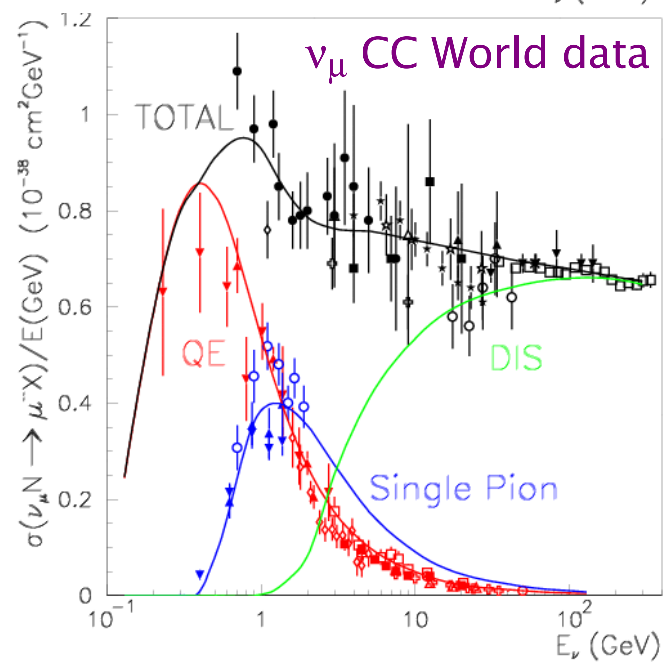




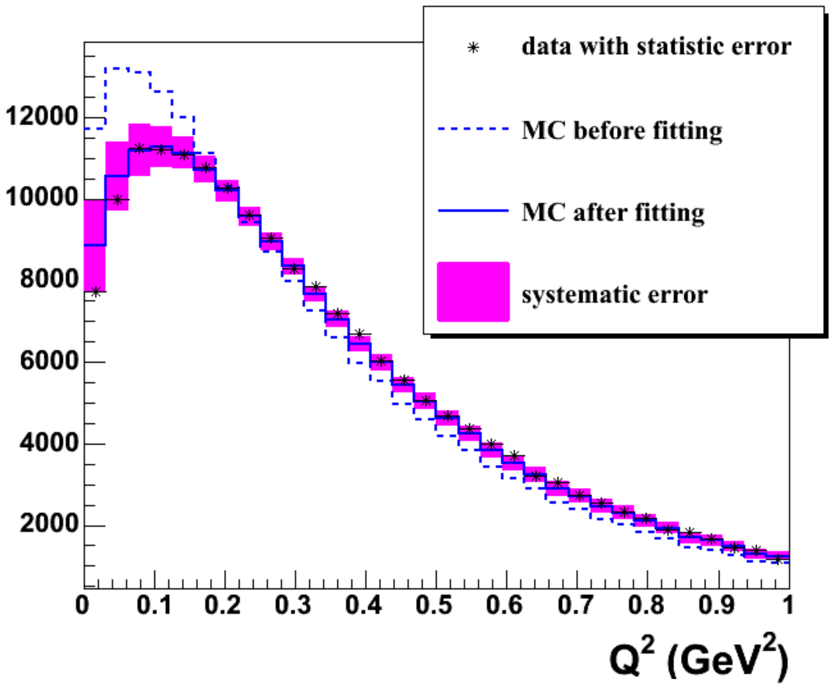




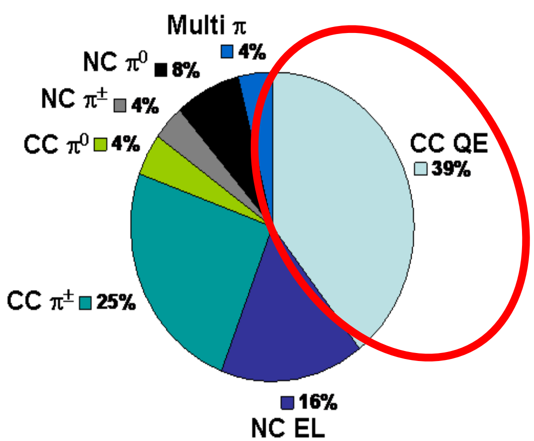


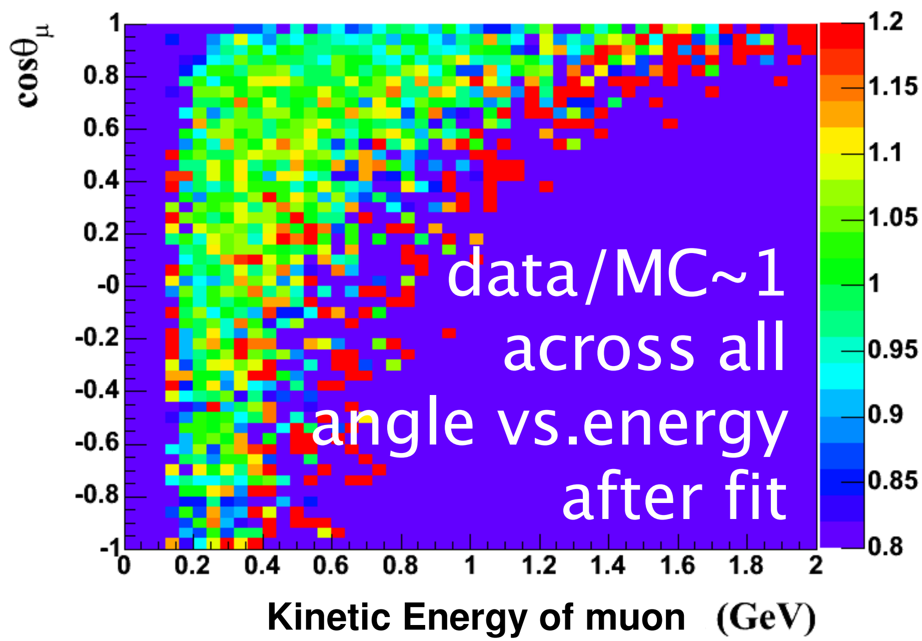


Tuning Nuance on internal ν_μ CCQE data



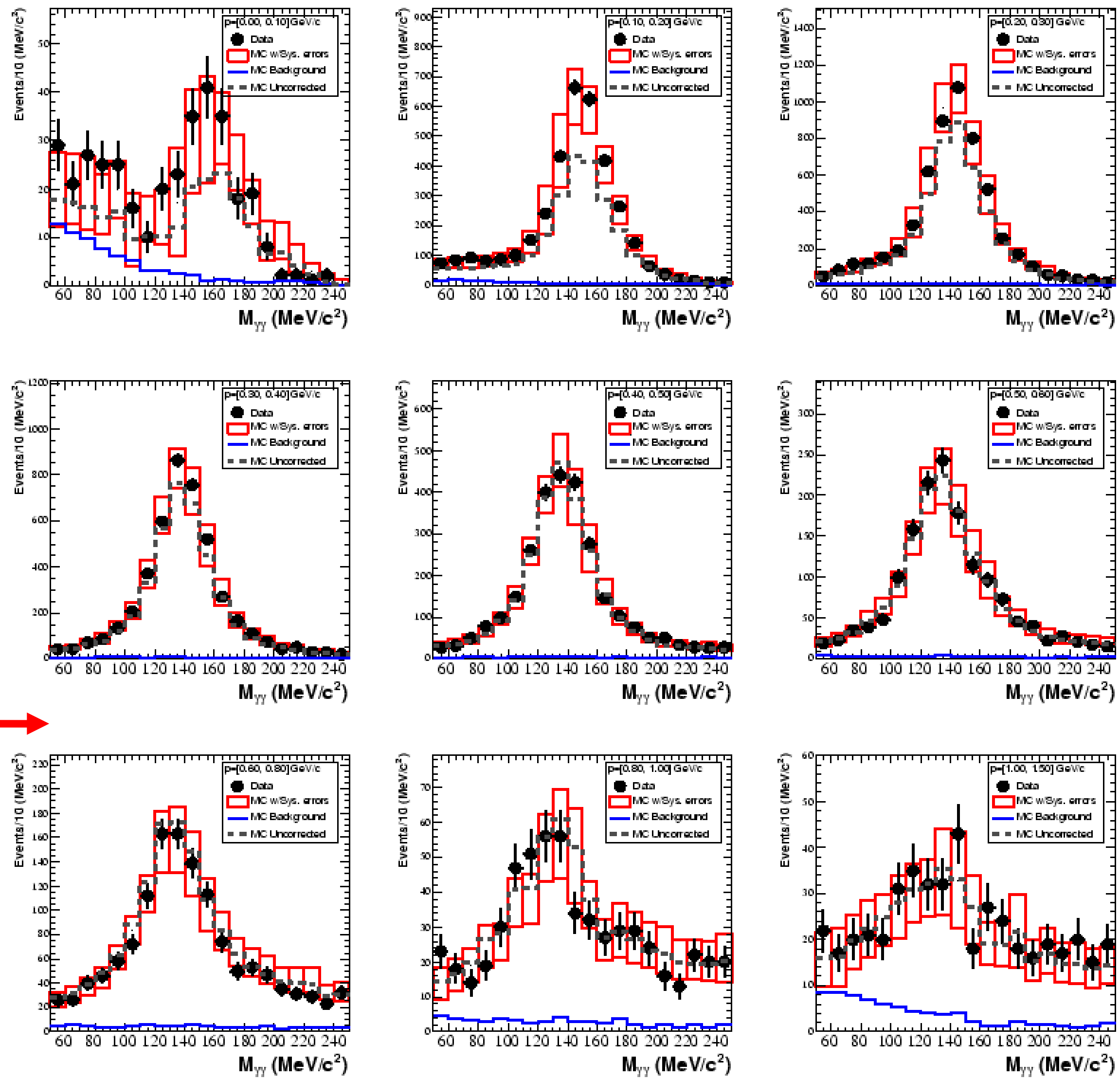
- From Q^2 fits to MB ν_μ CCQE data:
  - M_A^{eff} -- effective axial mass
  - E_{lo}^{SF} -- Pauli Blocking parameter
- From electron scattering data:
  - E_b -- binding energy
  - p_f -- Fermi momentum
- Model describes CCQE ν_μ data well



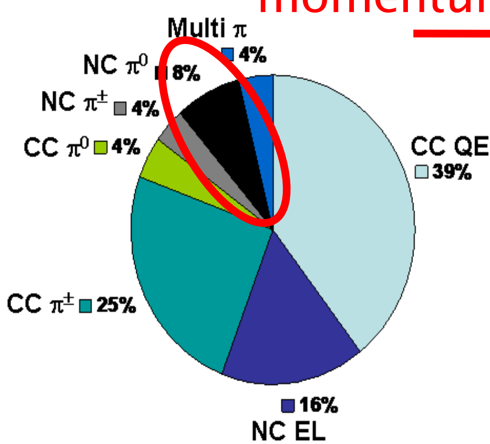


Tuning Nuance on internal NC π^0 data

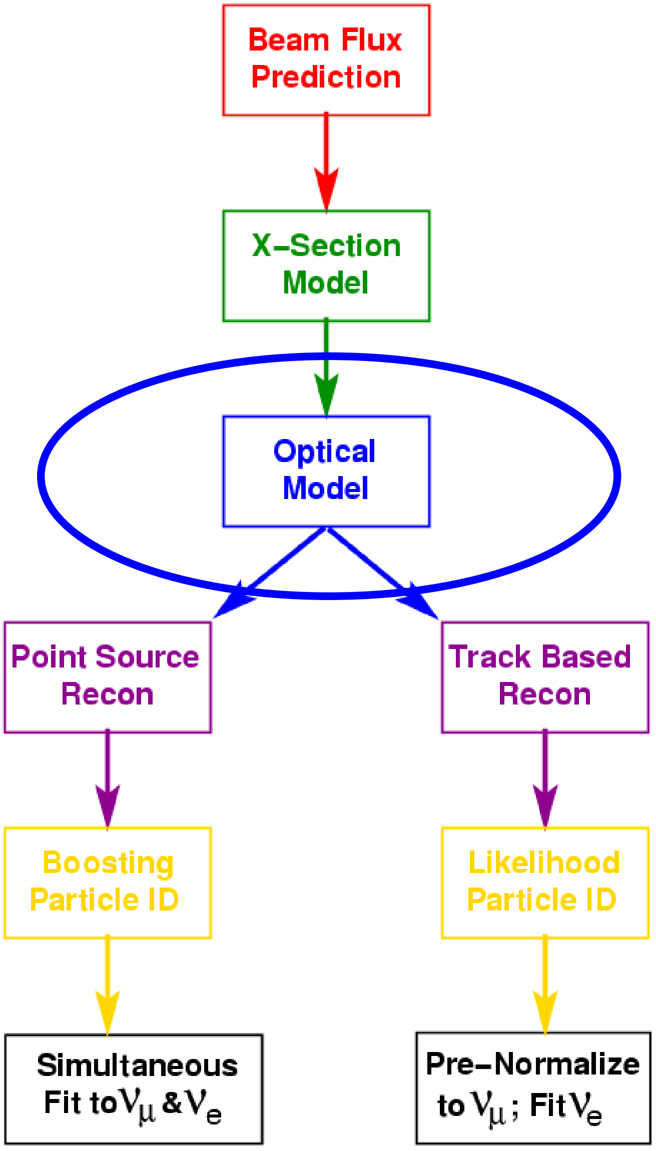
- 90%+ pure π^0 sample (mainly $\Delta \to N\pi^0$)
- Measure rate as function of momentum
- Default MC underpredicts rate at low momentum
- analysis reaches 1.5 GeV
- $\Delta \to N\gamma$ also constrained (though to a lesser extent)



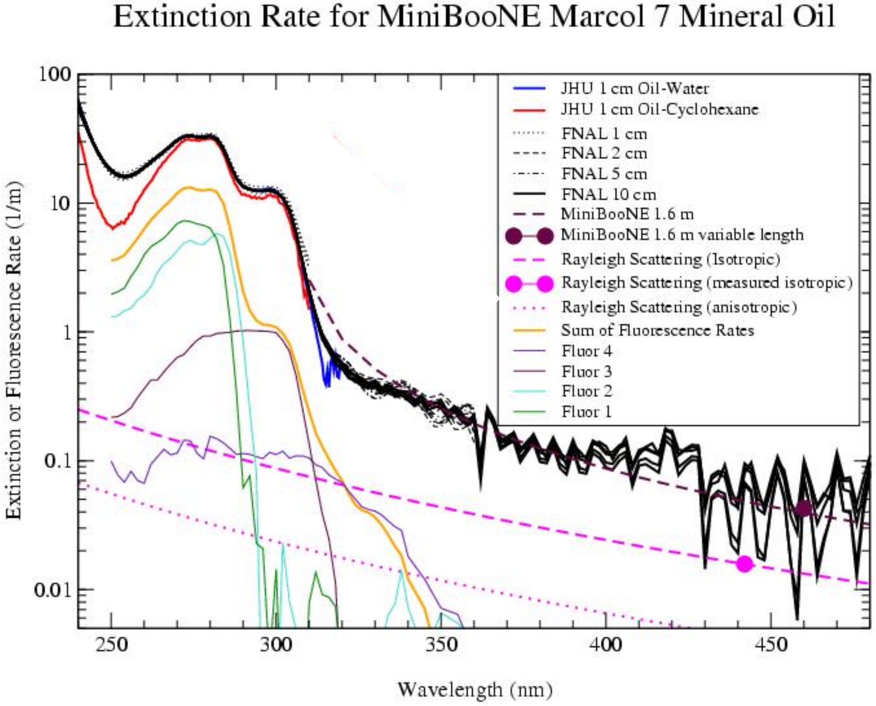
Invariant mass distributions in momentum bins

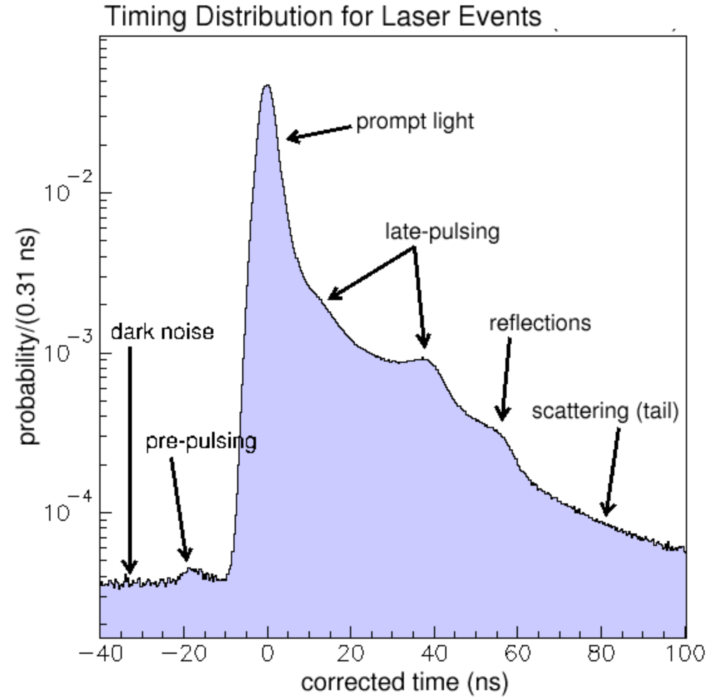


Optical Model



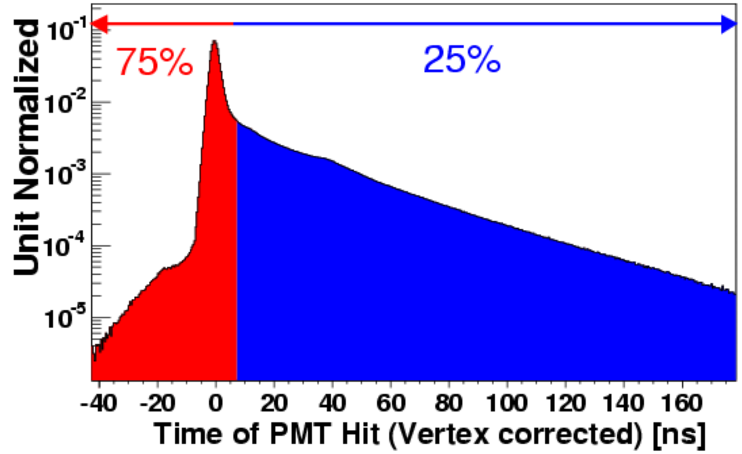
Light propagation in the detector





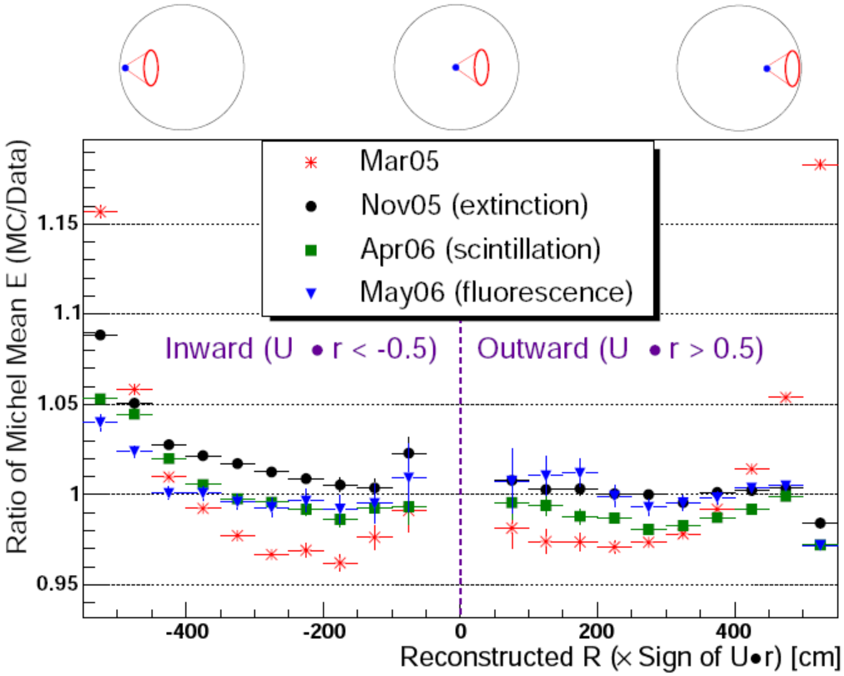
- Optical model is very complex
  - Cerenkov, scintillation, fluorescence
  - PMT Q/t response
  - Scattering, reflection, prepulses
- Overall, about 40 non-trivial parameters

Michel electron t distribution

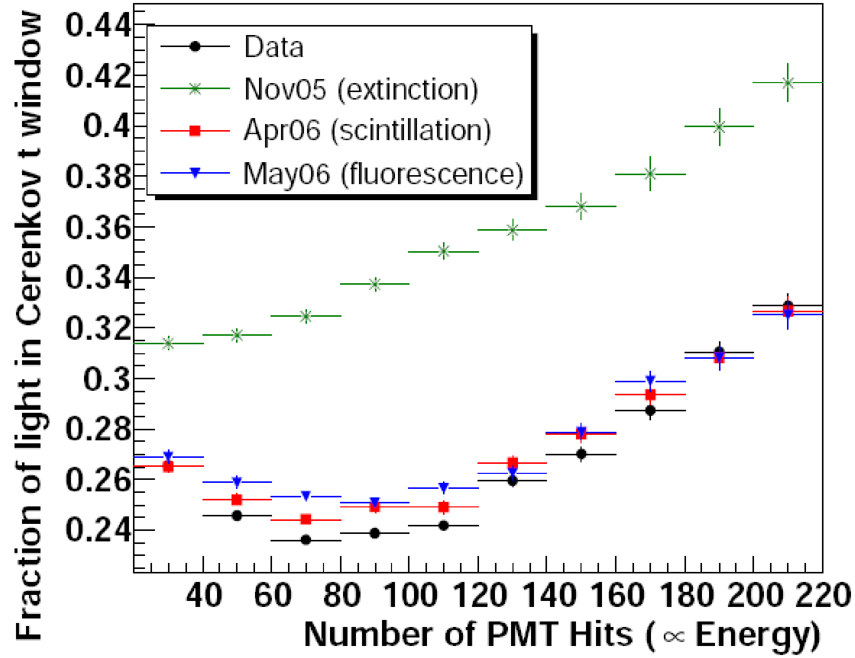


Tuning the optical model

Using Michel electrons...



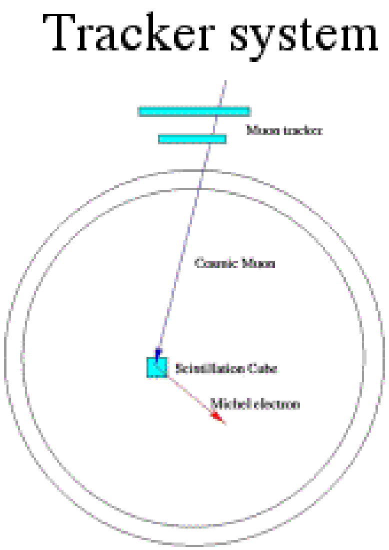
Using NC elastic ν interactions...

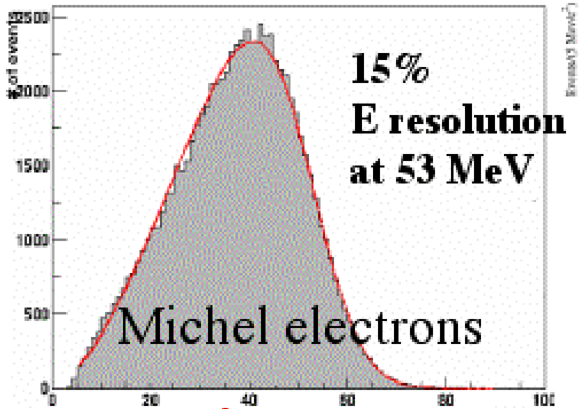


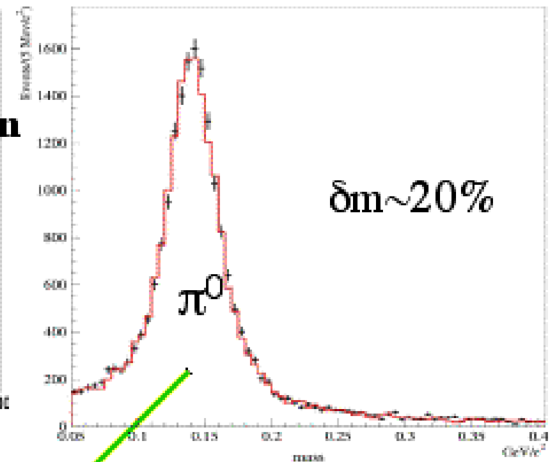
- Initial optical model defined through many benchtop measurements
- Subsequently tuned with *in situ* sources, examples
  - Left: Michel e populate entire tank, useful for tuning extinction
  - Right: NC elastic n interactions below Cerenkov threshold useful for distinguishing scintillation from fluorescence

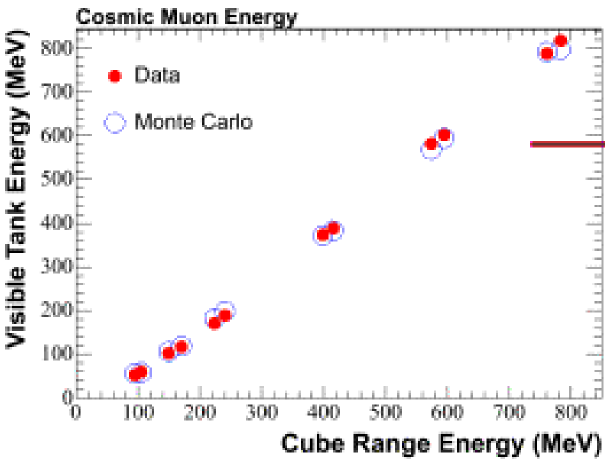
Calibration sources span various energies

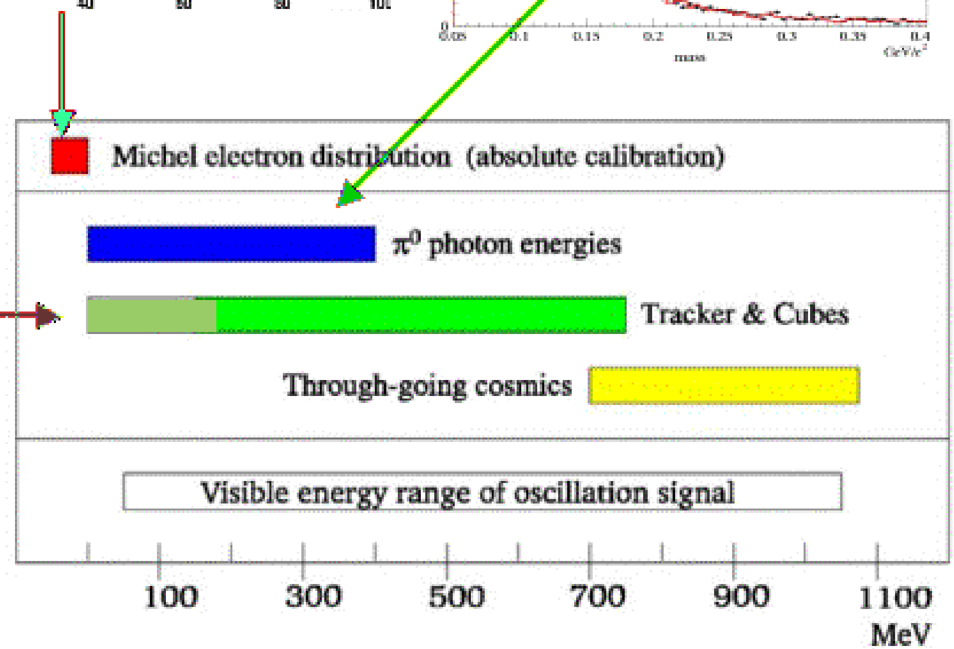
Calibration Sources



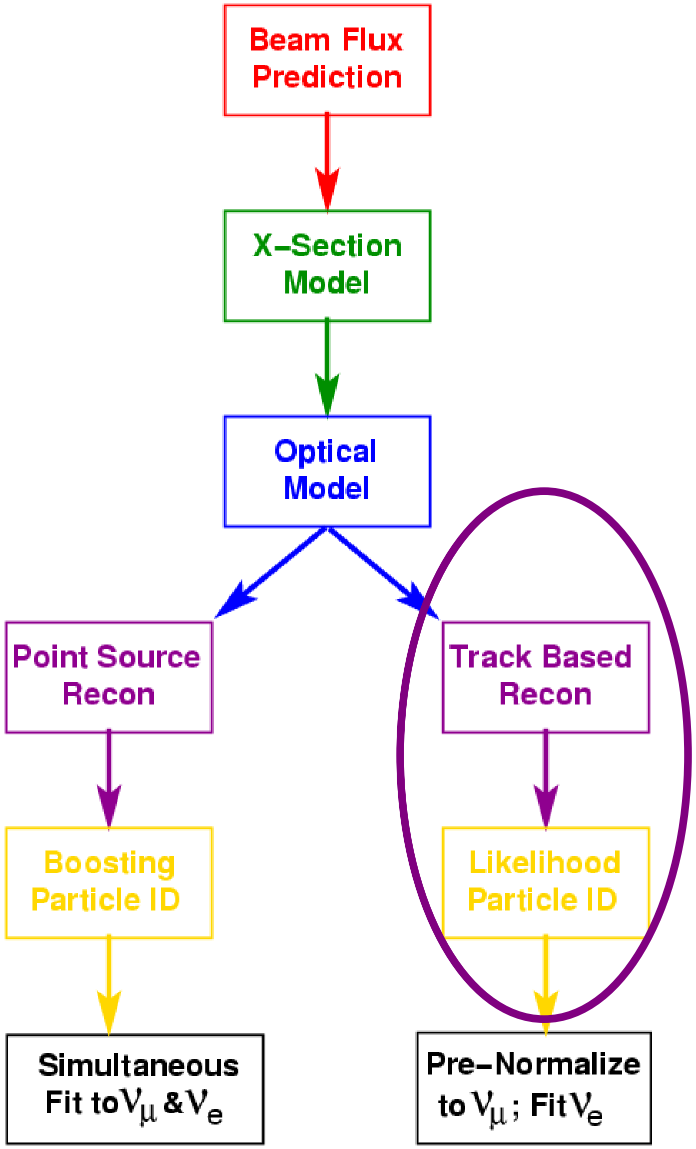






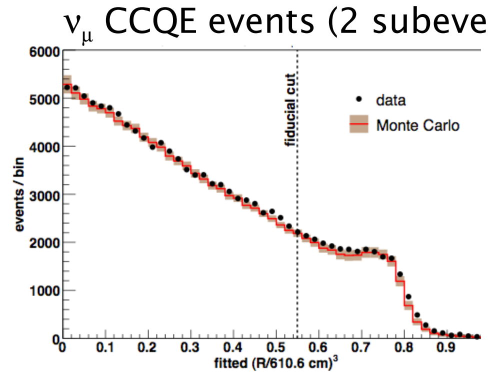


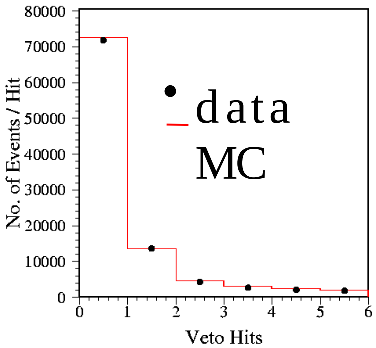
Track–Based Likelihood (TBL) Reconstruction and Particle ID

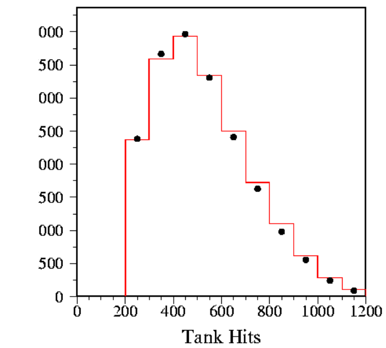


TBL Analysis: Separating e from μ

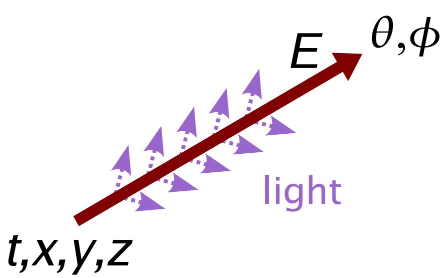
- Analysis pre-cuts
  - Only 1 subevent
  - Veto hits < 6
  - Tank hits > 200
  - Radius < 500 cm

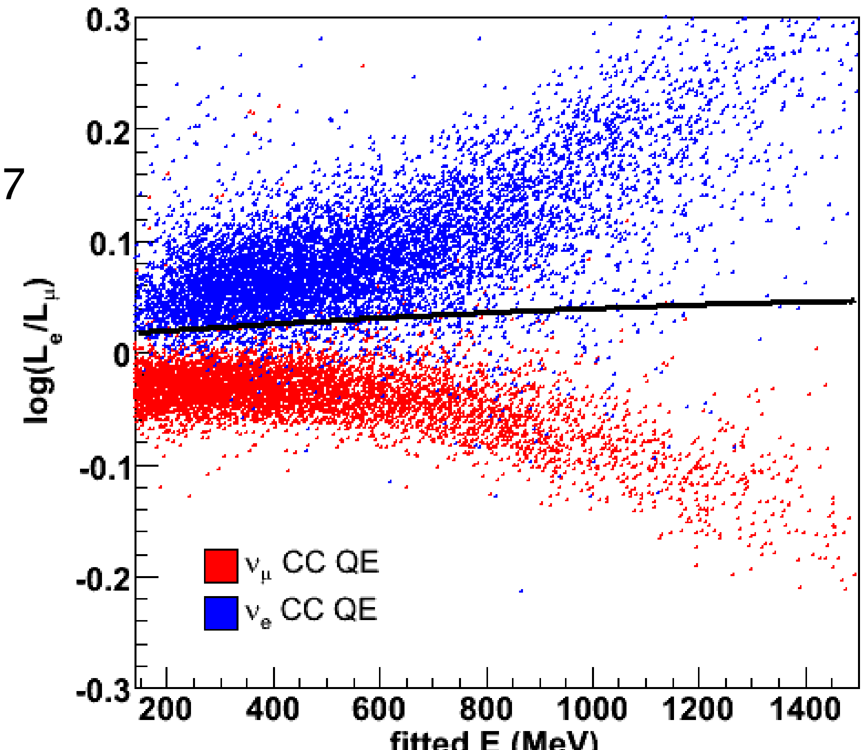






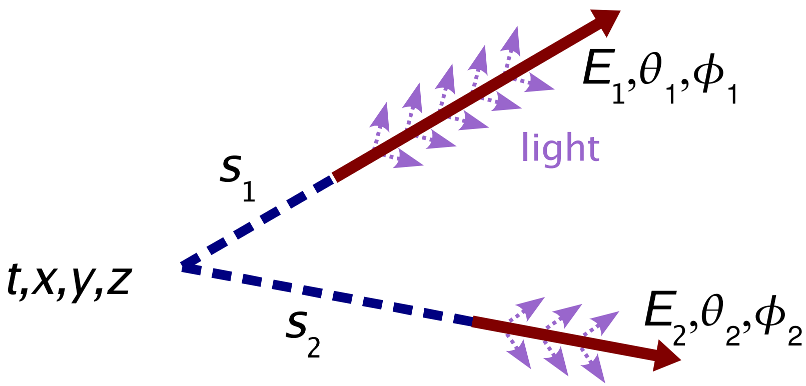
- Event is a collection of PMT-level info (q,t,x)
- Form sophisticated Q and T pdfs, and fit for 7 track parameters under 2 hypotheses
  - The track is due to an electron
  - The track is coming from a muon

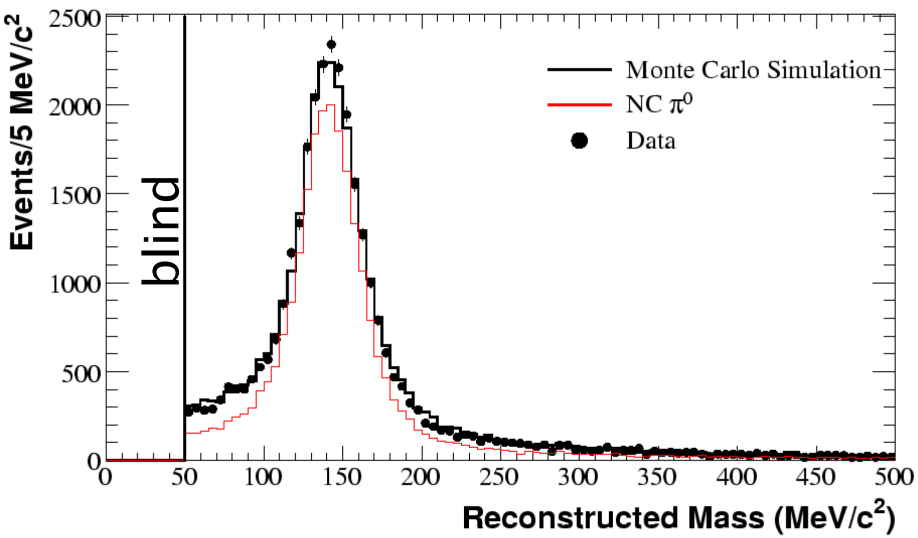


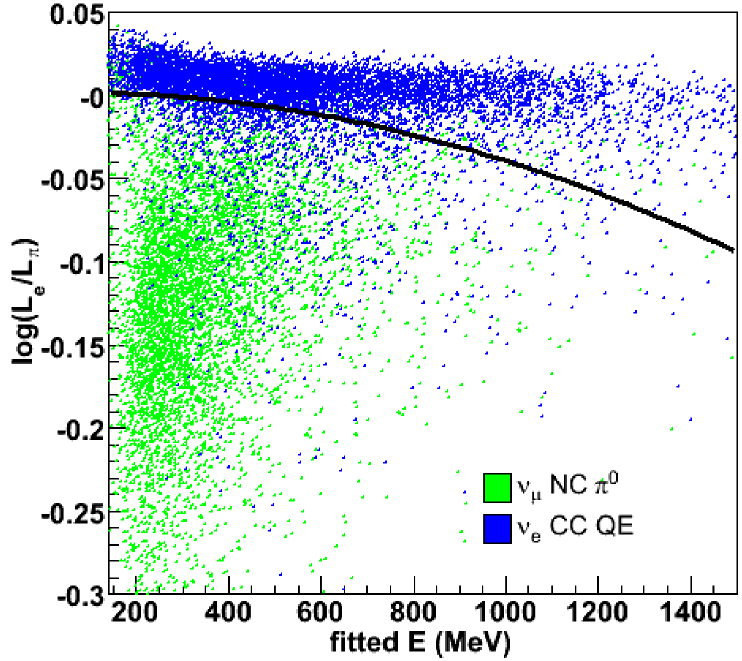


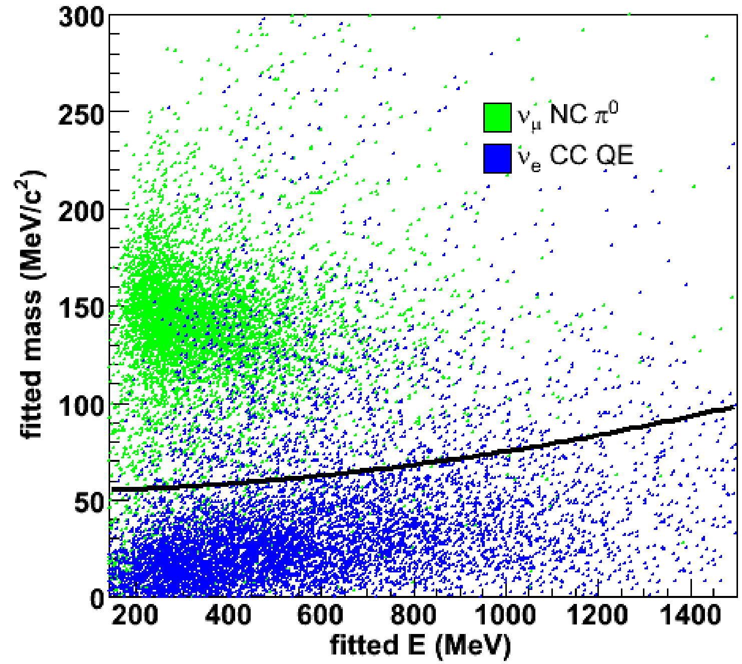
Separating e from π^0

- Extend fit to include two e-like tracks
- Very tenacious fit...8 minutes per event
- Nearly 500k CPU hours used

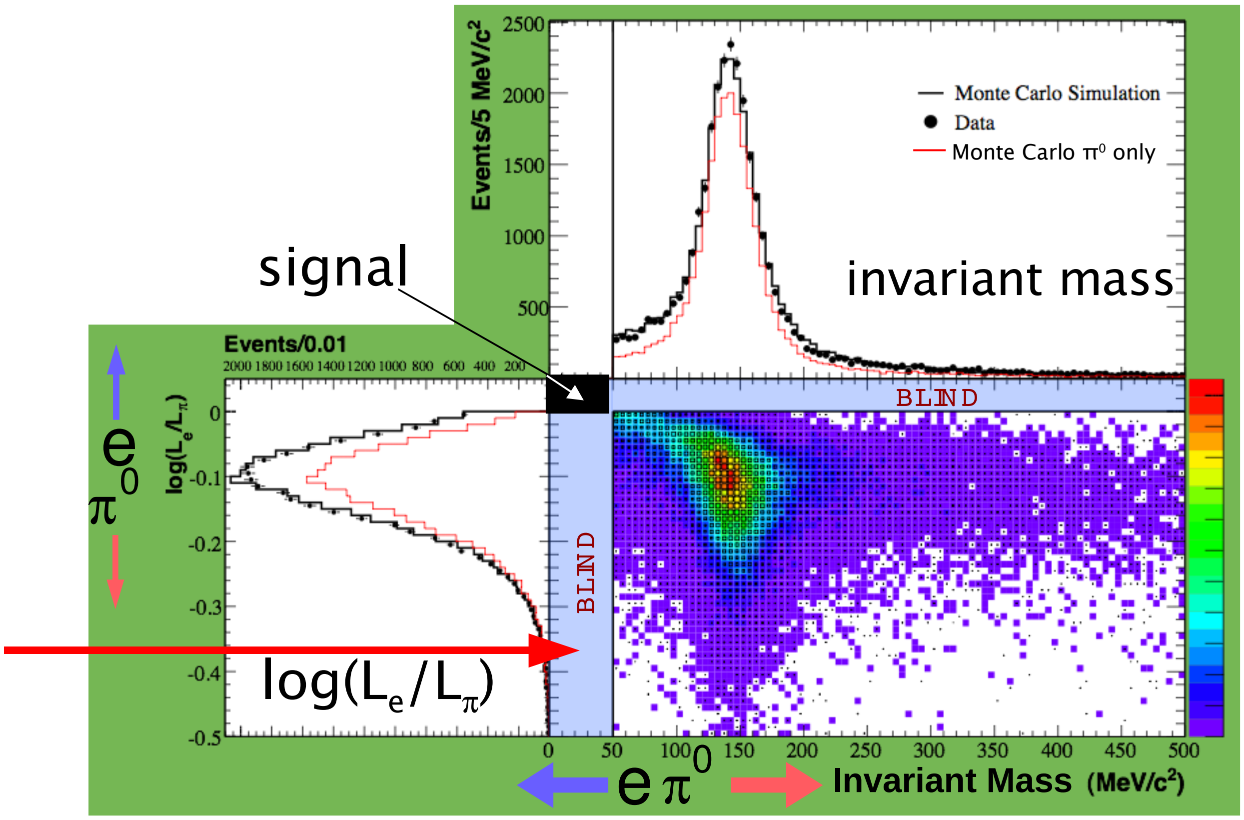






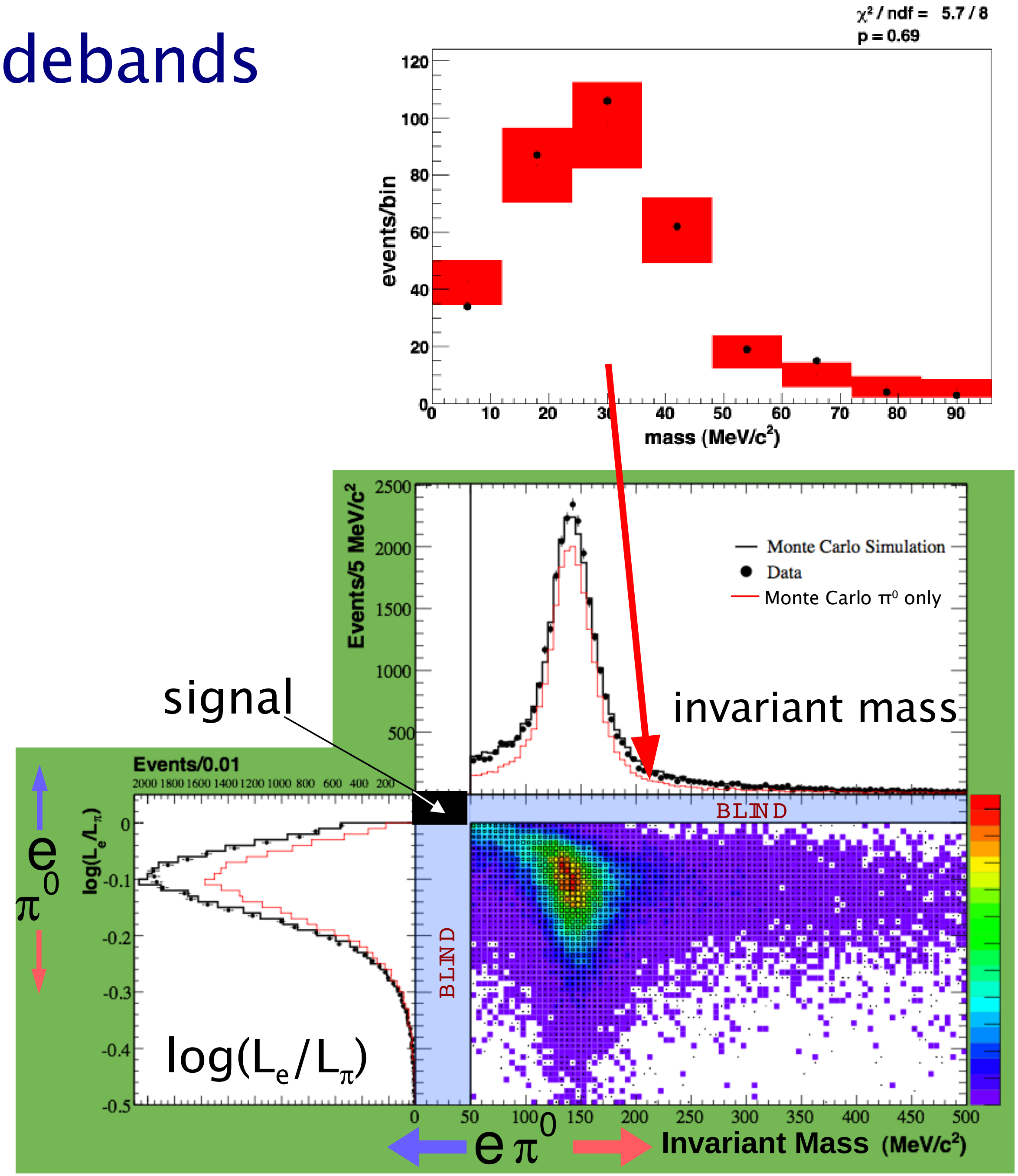


Checking signal sidebands



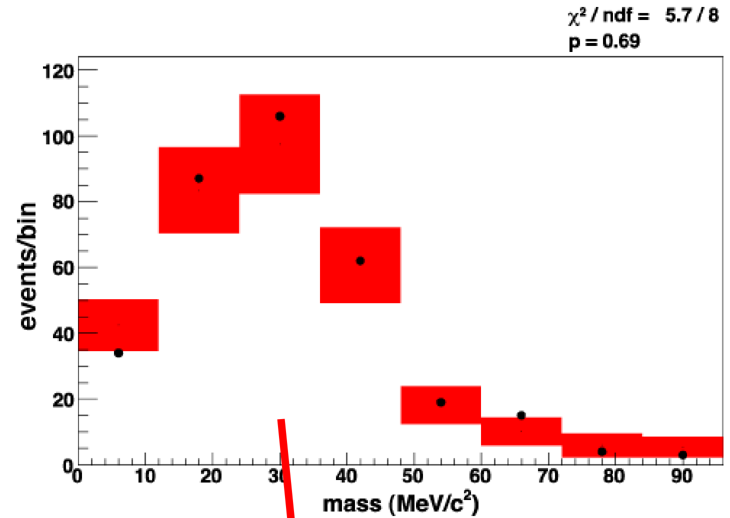
Checking signal sidebands

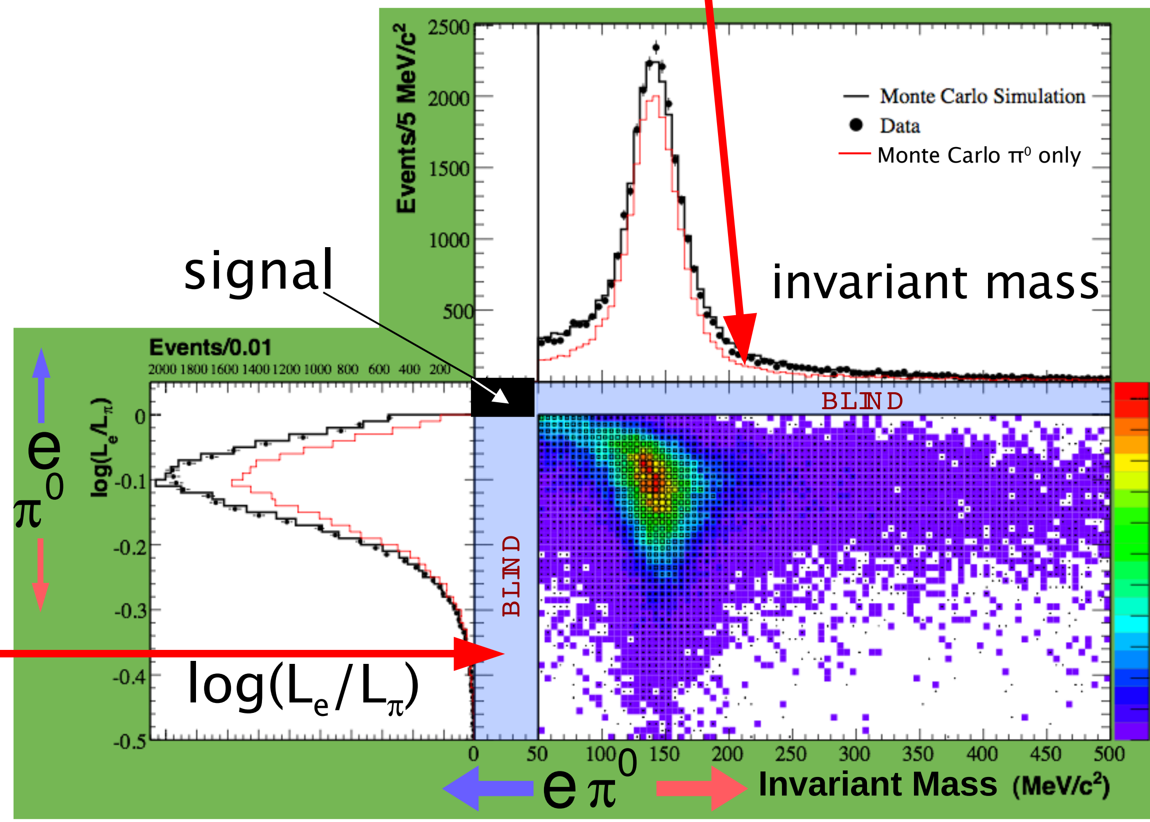
- Region at low $\log(L_e/L_\pi)$

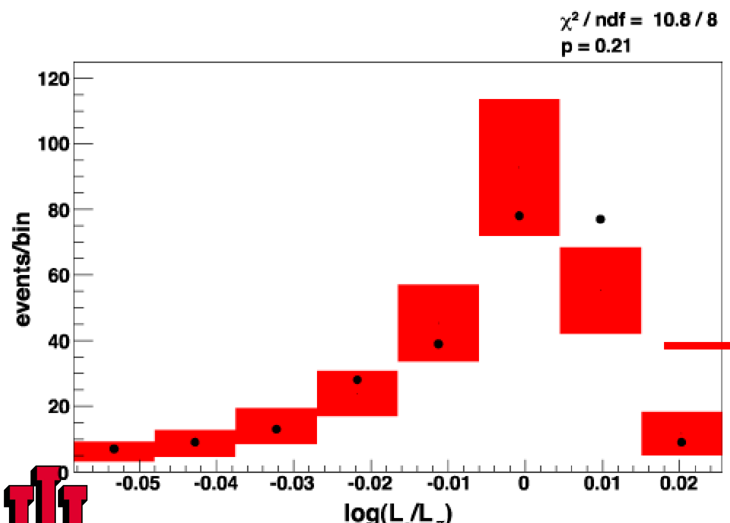


Checking signal sidebands

- Region at low $\log(L_e/L_\pi)$
- Region at low invariant mass

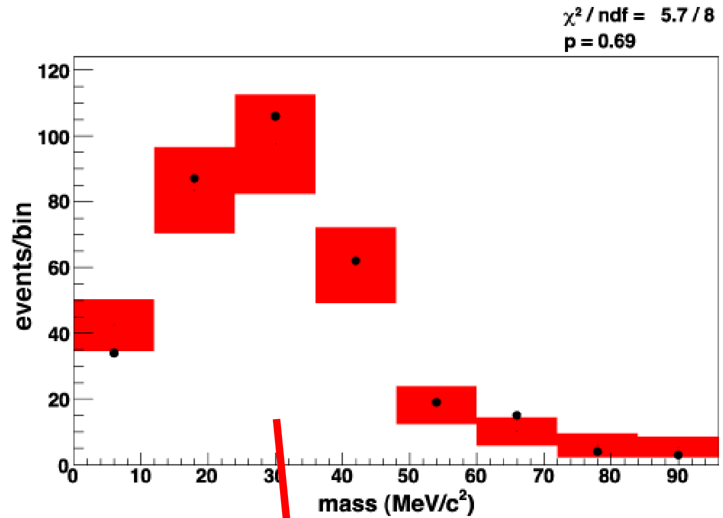


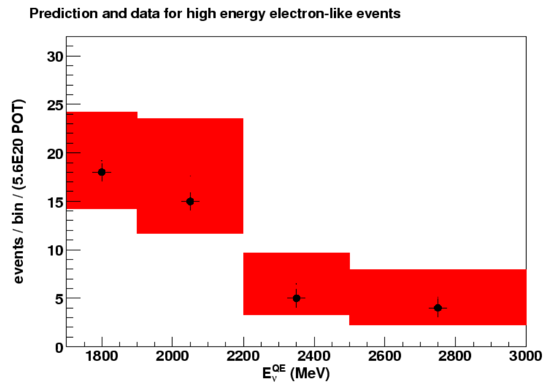


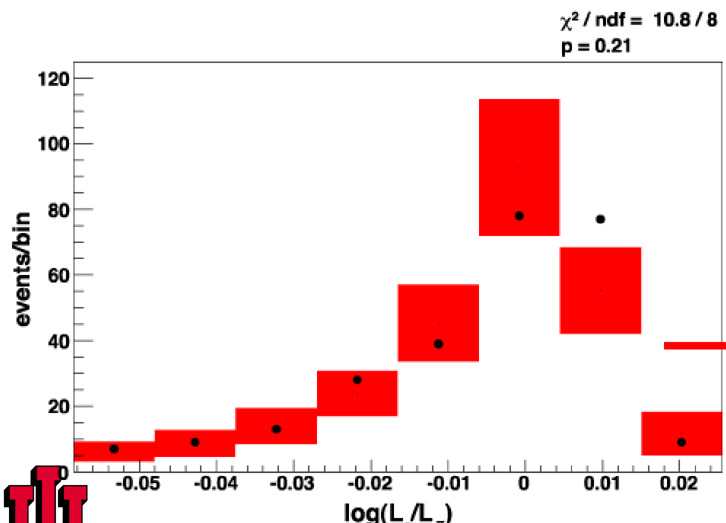


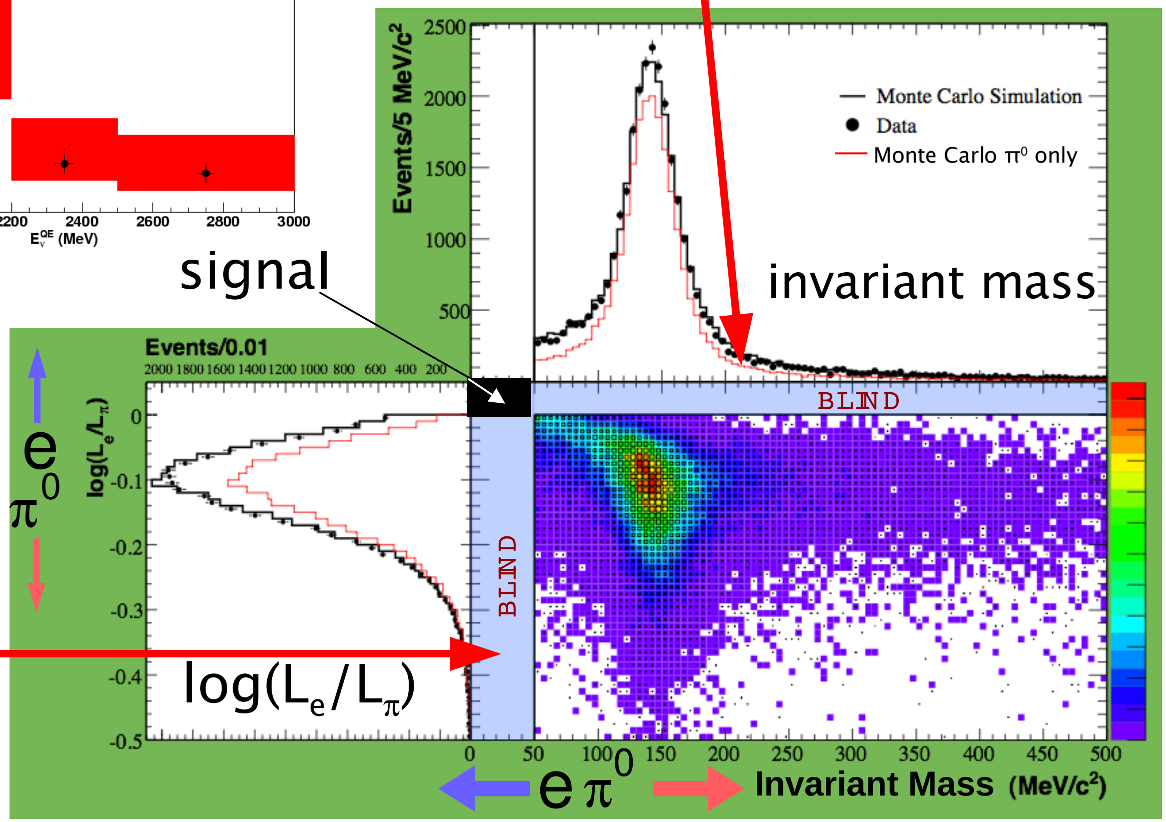
Checking signal sidebands

- Region at low $\log(L_e/L_\pi)$
- Region at low invariant mass
- Region in signal, but at high E_ν

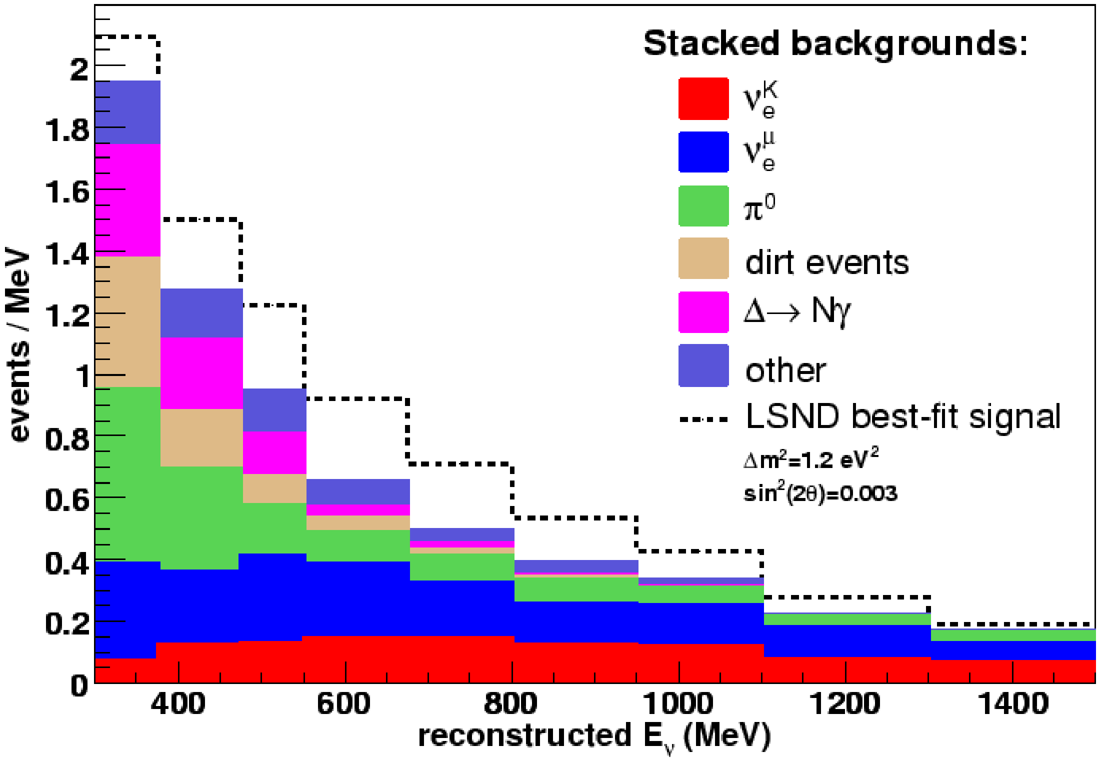








TBL Analysis: Expected event totals

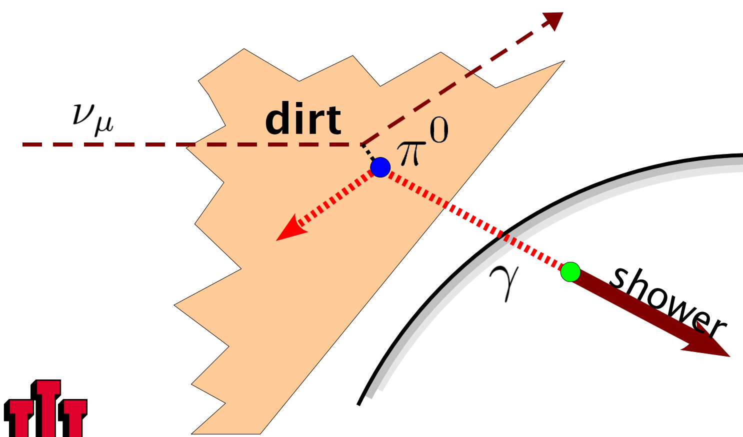


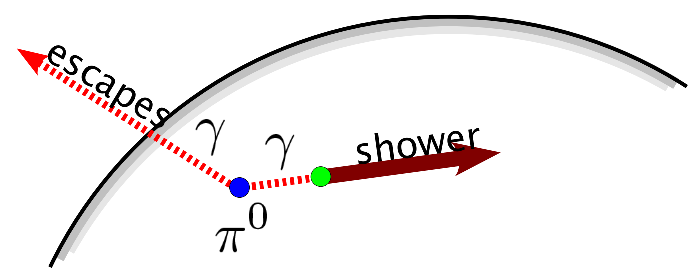
475 MeV – 1250 MeV

ν_e^K	94
ν_e^μ	132
π^0	62
dirt	17
$\Delta \rightarrow N\gamma$	20
other	33
total	358

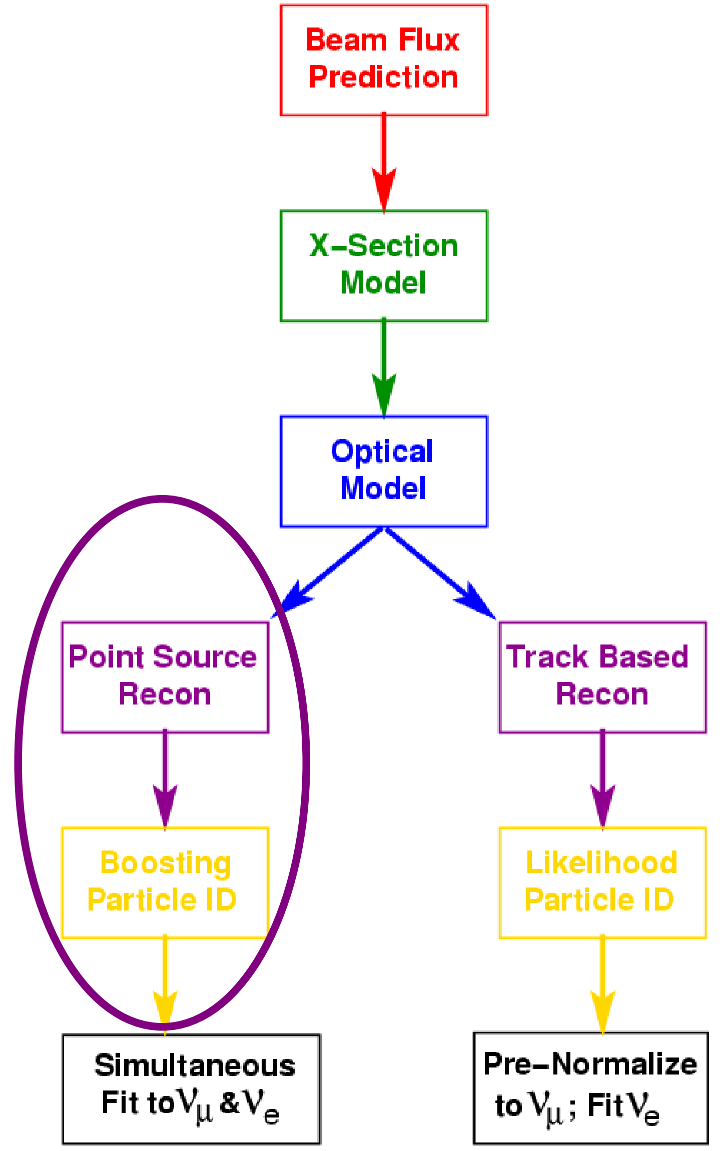
LSND best-fit $\nu_\mu \rightarrow \nu_e$ 126

S/sqrt(B)=6.8





Boosted Decision Tree (BDT) Reconstruction and Particle ID



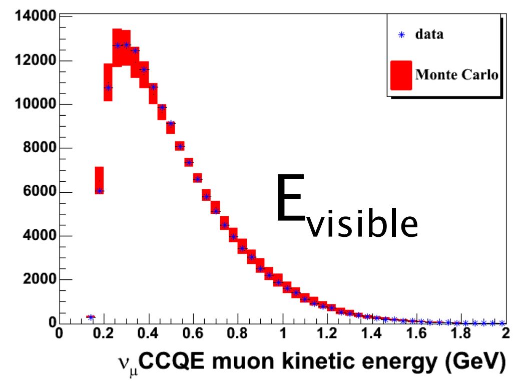
BDT Reconstruction

- Same pre-cuts as TBL (taking R from different reconstruction)
- Different reconstruction:
  - Treats particles more like point sources, *i.e.* not as careful about dE/dx
  - Not as tenacious about getting out of local minima, particularly with pion fit
  - Reconstruction runs nearly 10 times faster
- To make up for the simple fit, the BDT analysis relies on a form of machine learning, the boosted decision tree. Byron P. Roe, *et al.*, NIM A543 (2005) 577.

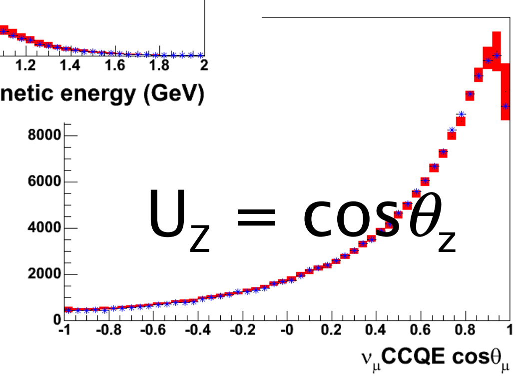
BDT Resolution:
vertex: 24 cm
direction: 3.8°
energy 14%

TBL Resolution:
vertex: 22 cm
direction: 2.8°
energy 11%

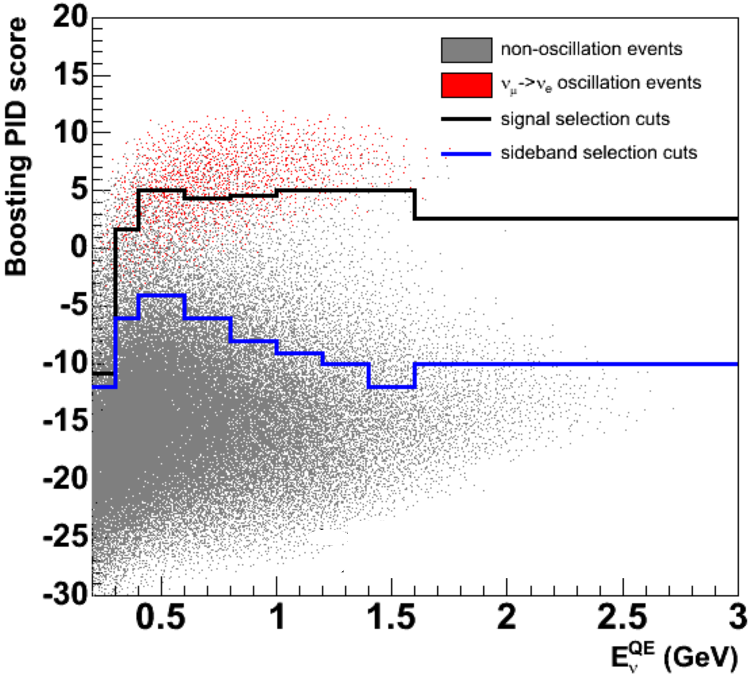
- Boosting Input Variables:
  - Low-level (# tank hits, early light fraction, etc.)
  - High-level (Q2, U_z, fit likelihoods, etc.)
  - Topology (charge in anuli, isotropic light, etc.)
- A total of 172 variables were used
- All 172 were checked for agreement within errors in 5 important 'boxes' (ν_μ CCQE, NC π^0, NC-elastic, Michel decay e, 10% closed)
- Boosting Output: Single 'score', + is signal-like



ν_μ CCQE Examples

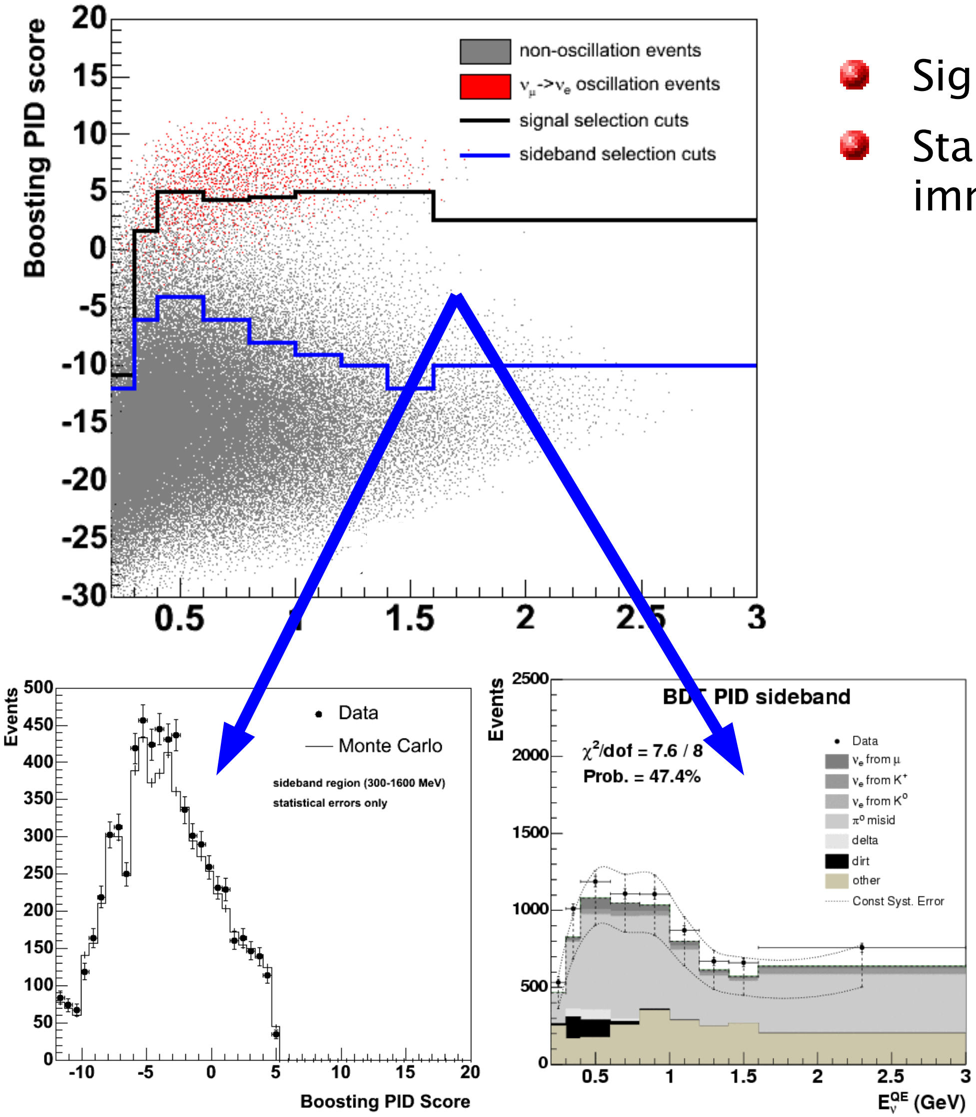


BDT Analysis: Signal/background regions



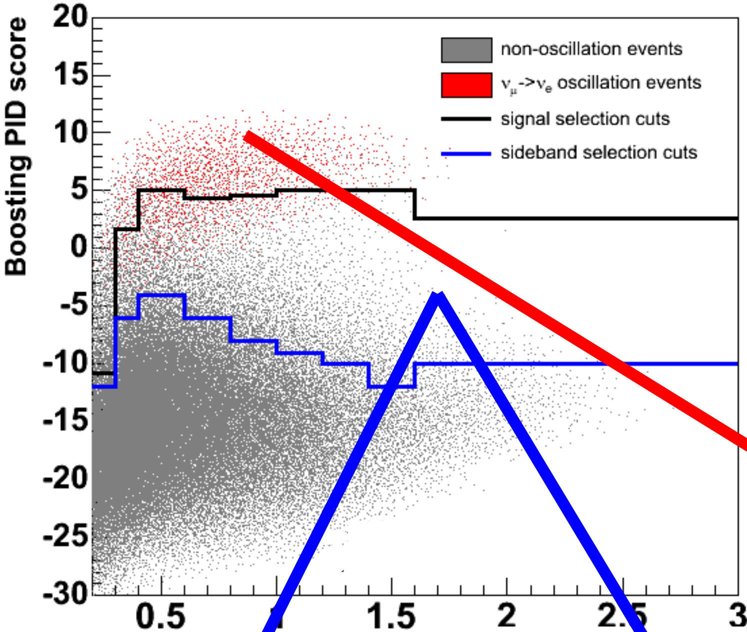
- Signal prediction (red) versus all bkgs (gray)

BDT Analysis: Signal/background regions



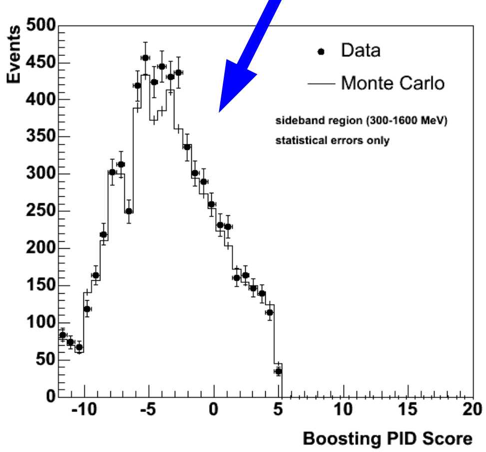
- Signal prediction (red) versus all bkgs (gray)
- Start by looking at data in 'sideband'...region immediately adjacent to signal region

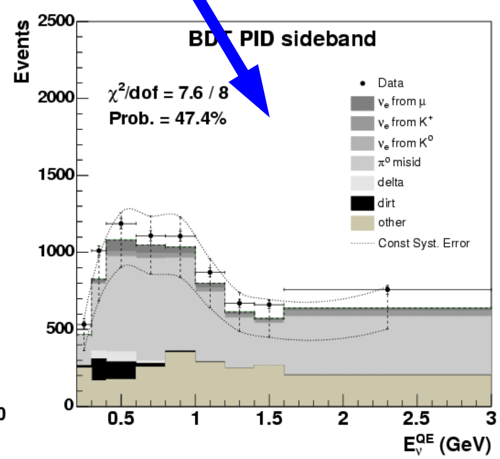
BDT Analysis: Signal/background regions

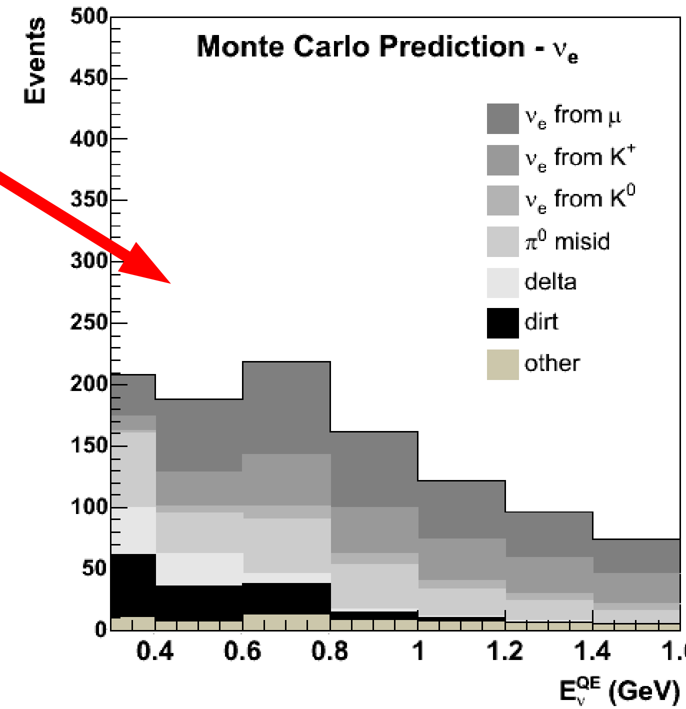


- Signal prediction (red) versus all bkgs (gray)
- Start by looking at data in 'sideband'...region immediately adjacent to signal region
- Satisfied with agreement? Finalize background prediction
- In 500-1200 MeV range: 603 bkg, LSND best-fit $\nu_\mu \to \nu_e$ 203

S/sqrt(B)=8.3



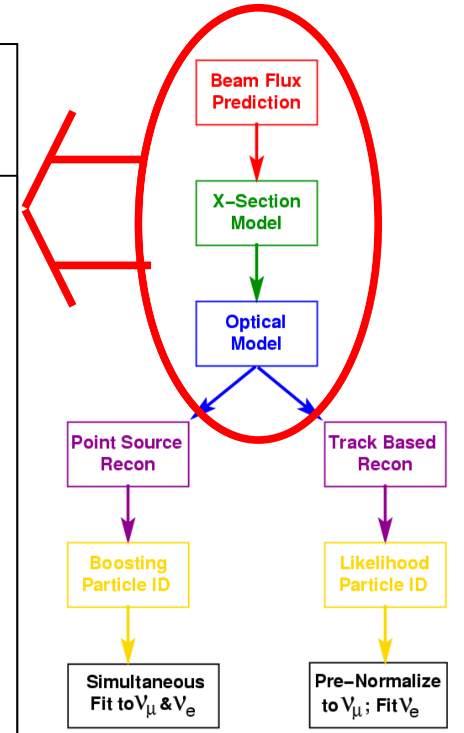




Systematic Error Analysis and Results

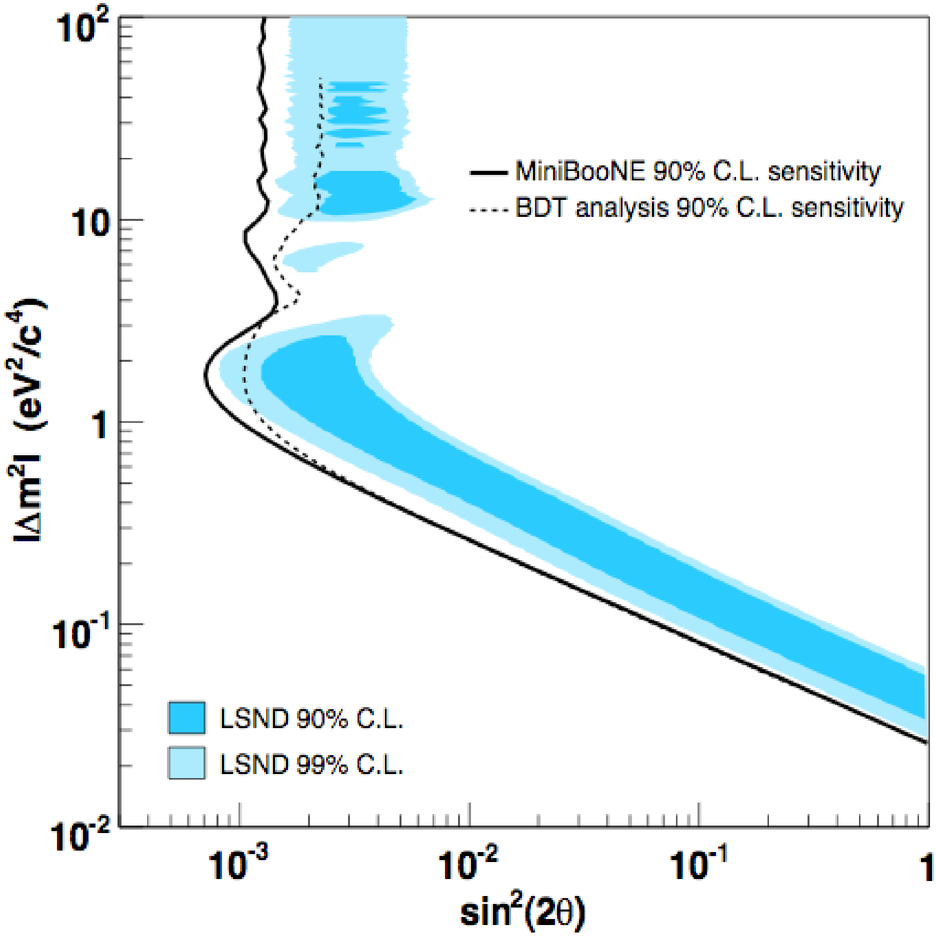
Final error budget (diagonals only...greatly simplified)

Source of uncertainty on ν_e background	TBL/BDT error in %	Constrained by MB data	Reduced by tying ν_e to ν_μ
Flux from π^+/μ^+ decay	6.2 / 4.3	√	√
Flux from K^+ decay	3.3 / 1.0	√	√
Flux from K^0 decay	1.5 / 0.4	√	√
Target/beam models	2.8 / 1.3	√	
ν–cross section	12.3 / 10.5	√	√
NC π^0 yield	1.8 / 1.5	√	
Dirt interactions	0.8 / 3.4	√	
Optical model	6.1 / 10.5	√	√
DAQ electronics model	7.5 / 10.8	√	



- Every checkmark in this table could easily consume a 30 minute talk
  - All error sources had some *in situ* constraint
  - Some reduced by combined fit to ν_μ and ν_e
- Errors arise from common uncertainties in flux, xsec, and optical model
- Reconstruction and PID unique
  - BDT had higher signal–to–background
  - TBL more impervious to systematics
  - About 50% event overlap

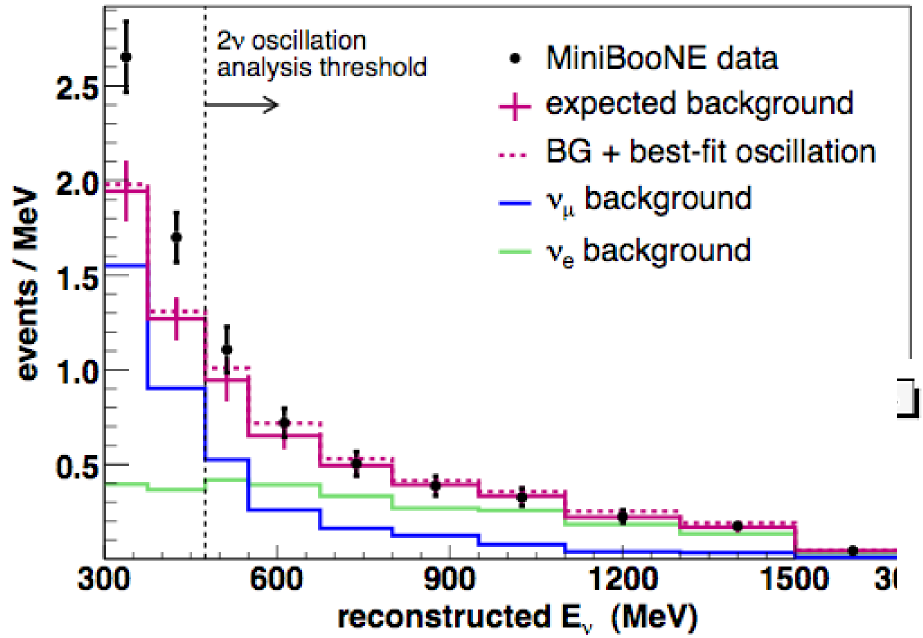
BDT/TBL sensitivity comparison



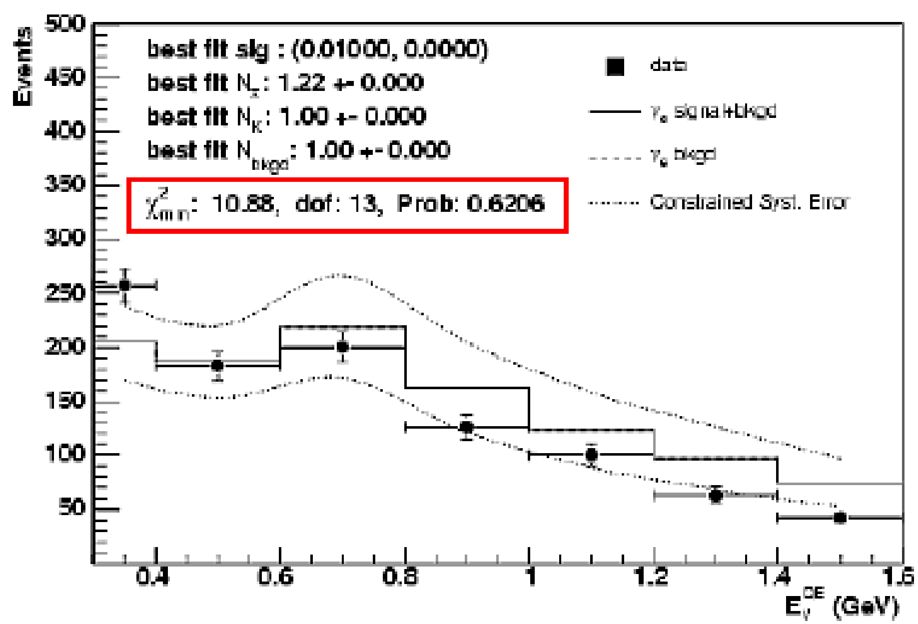
- Sensitivity is determined from simulation only (no data yet!)
- Decided before unblinding that the analysis with higher sensitivity would be the final analysis
- TBL (solid) is better at high Δm^2
- 90% CL defined by $\Delta\chi^2 = 1.64$

After many man-years and CPU-hours...

Finally we see the data in the signal region...



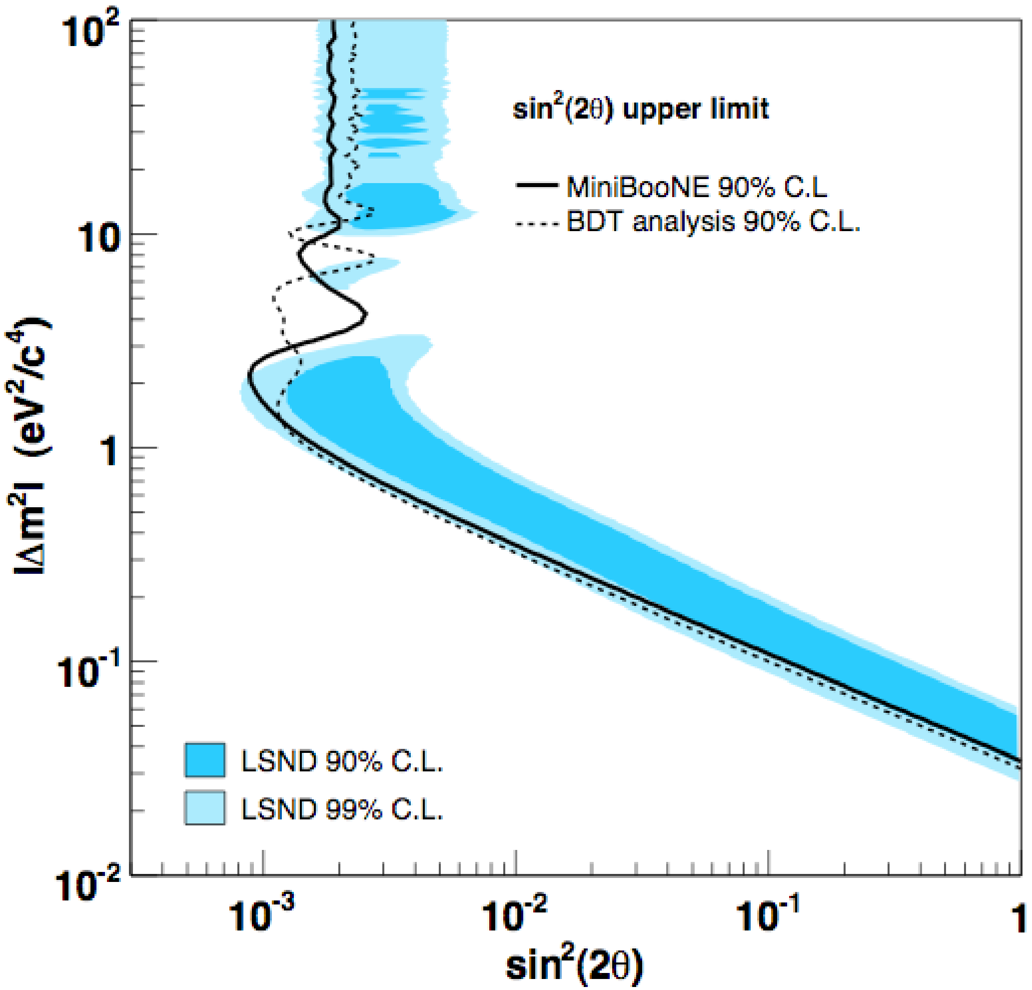
- TBL shows no sign of an excess in the analysis region (where the LSND signal is expected for the 2ν mixing hypothesis)
- Visible excess at low E



- BDT has a good fit and no sign of an excess, in fact the data is low relative to the prediction
- Also sees an excess at low E, but larger normalization error complicates interpretation

Neither analysis shows an evidence for $\nu_\mu \rightarrow \nu_e$ appearance in the analysis region

Fit results mapped into $\sin^2(2\theta)$ Δm^2 plane

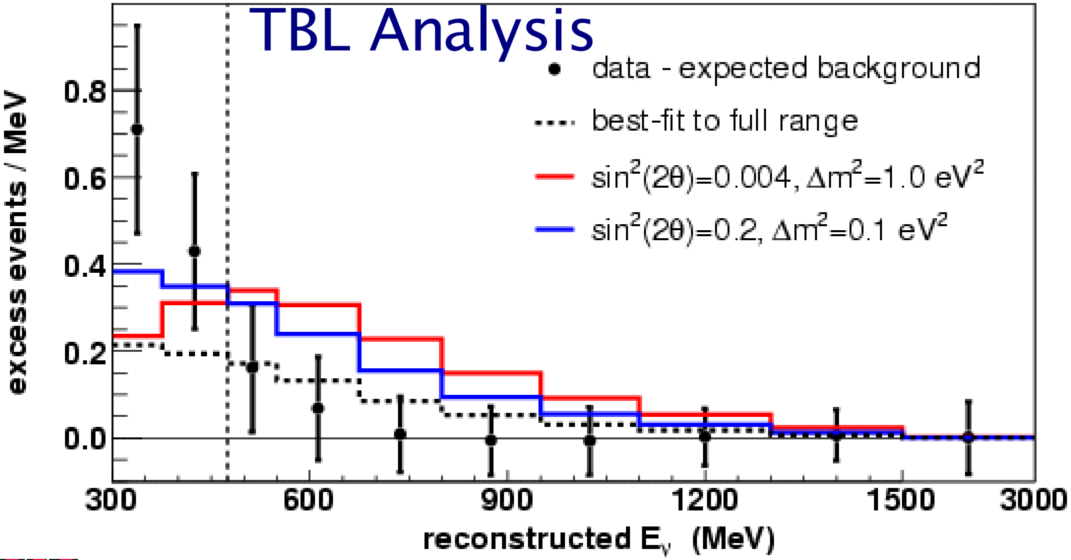


- Energy-fit analysis:
  - solid: TBL
  - dashed: BDT
- Independent analyses in good agreement
- Looks similar to sensitivity because of the lack of a signal
- Had there been a signal, these curves would have curled around and closed into contours
- MiniBooNE and LSND incompatible at a 98% CL for all Δm^2 under a 2ν mixing hypothesis.

Future work for MiniBooNE

- Papers in support of this analysis
  - NC π^0 background measurement
  - ν_μ CCQE analysis
- Continued improvements of the ν oscillation analysis
  - Combined BDT and TBL
  - More work on reducing systematics
- Re-examine low E backgrounds and significance of low E excess

TBL Analysis



- Lots of work on cross-sections
- MB has more ν_μ interactions than prior experiments in an energy range useful to future ν expts.
- Event counts before cuts:

ν channel	events
all channels	810k
CC quasielastic	340k
NC elastic	150k
CC π^+	180k
CC π^0	30k
NC π^0	48k
NC $\pi^{+/-}$	27k
CC/NC DIS, multi-π	35k

6×10^{20} POT
ν mode

$\bar{\nu}$ channel	events
all channels	54k
CC quasielastic	24k
NC elastic	10k
CC π^-	8.9k
CC π^0	1.7k
NC π^0	4.9k
NC $\pi^{+/-}$	1.8k
CC/NC DIS, multi-π	1.9k

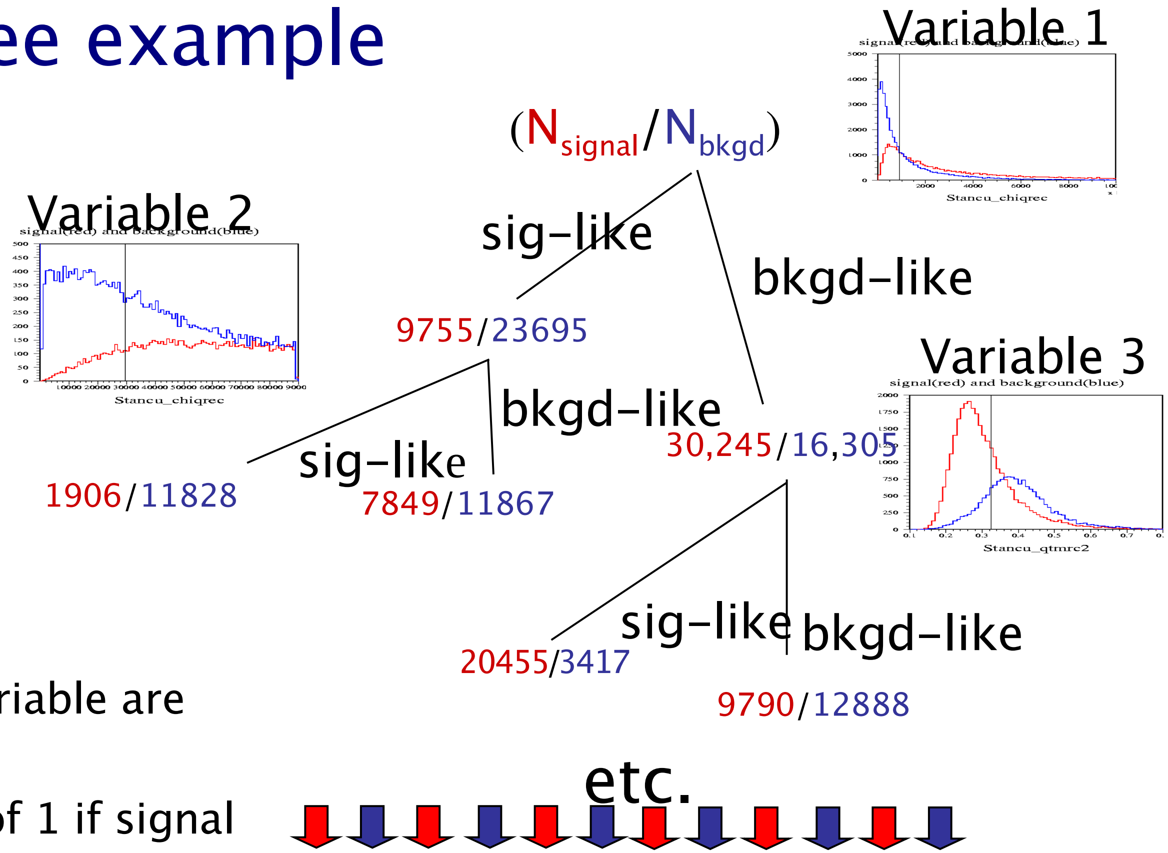
2×10^{20} POT
$\bar{\nu}$ mode

- Currently running in anti-ν mode for anti-ν cross sections

Backup Slides

Decision tree example

(sequential series of cuts
based on MC study)



This tree is one of many possibilities...

- Optimal cuts on each variable are determined
- An event gets a weight of 1 if signal −1 if background
- Hard to identify backgrounds are iteratively given more weight
- Many trees built
- PID 'score' established from ensemble

Handling uncertainties in the analysis

What we begin with...

For a given source
of uncertainty,

Errors on a wide range
of parameters
in the underlying model

... what we need

For a given source
of uncertainty,

Errors in bins of
E_ν^{QE}
and information on
the correlations
between bins

Incorporating the ν_μ constraint into the errors

Two Approaches

TBL: Reweight MC prediction to match measured ν_μ result
(accounting for systematic error correlations)

BDT: include the correlations of ν_μ to ν_e in the error matrix:

$$\chi^2 = \begin{pmatrix} \Delta_i^{\nu_e} & \Delta_i^{\nu_\mu} \end{pmatrix} \begin{pmatrix} M_{ij}^{e,e} & M_{ij}^{e,\mu} \\ M_{ij}^{\mu,e} & M_{ij}^{\mu,\mu} \end{pmatrix}^{-1} \begin{pmatrix} \Delta_j^{\nu_e} \\ \Delta_j^{\nu_\mu} \end{pmatrix}$$

where $\Delta_i^{\nu_e} = \mathrm{Data}_i^{\nu_e} - \mathrm{Pred}_i^{\nu_e}(\Delta m^2, \sin^2 2\theta)$ and $\Delta_i^{\nu_\mu} = \mathrm{Data}_i^{\nu_\mu} - \mathrm{Pred}_i^{\nu_\mu}$

Systematic (and statistical) errors are included in $(M_{ij})^{-1}$,
where *i, j* are bins of E_ν^{QE}

Example: Underlying X-section parameter errors

(Many are common to ν_μ and ν_e and cancel in the fit)

M_A^{QE}, e_{lo}^{sf}	6%, 2% (stat + bkg only)	determined from MiniBooNE ν_μ QE data
QE σ norm	10%	
QE σ shape	function of E_ν	
ν_e/ν_μ QE σ	function of E_ν	

NC π^0 rate	function of π^0 mom	determined from MiniBooNE ν_μ NC π^0 data
M_A^{coh}, coh σ	±25%	
$\Delta \to N\gamma$ rate	function of γ mom + 7% BF	

E_B, p_F	9 MeV, 30 MeV	determined from other experiments
Δs	10%	
$M_A^{1\pi}$	25%	
$M_A^{N\pi}$	40%	
DIS σ	25%	

Extracting the OM systematic error

- external measurements essential

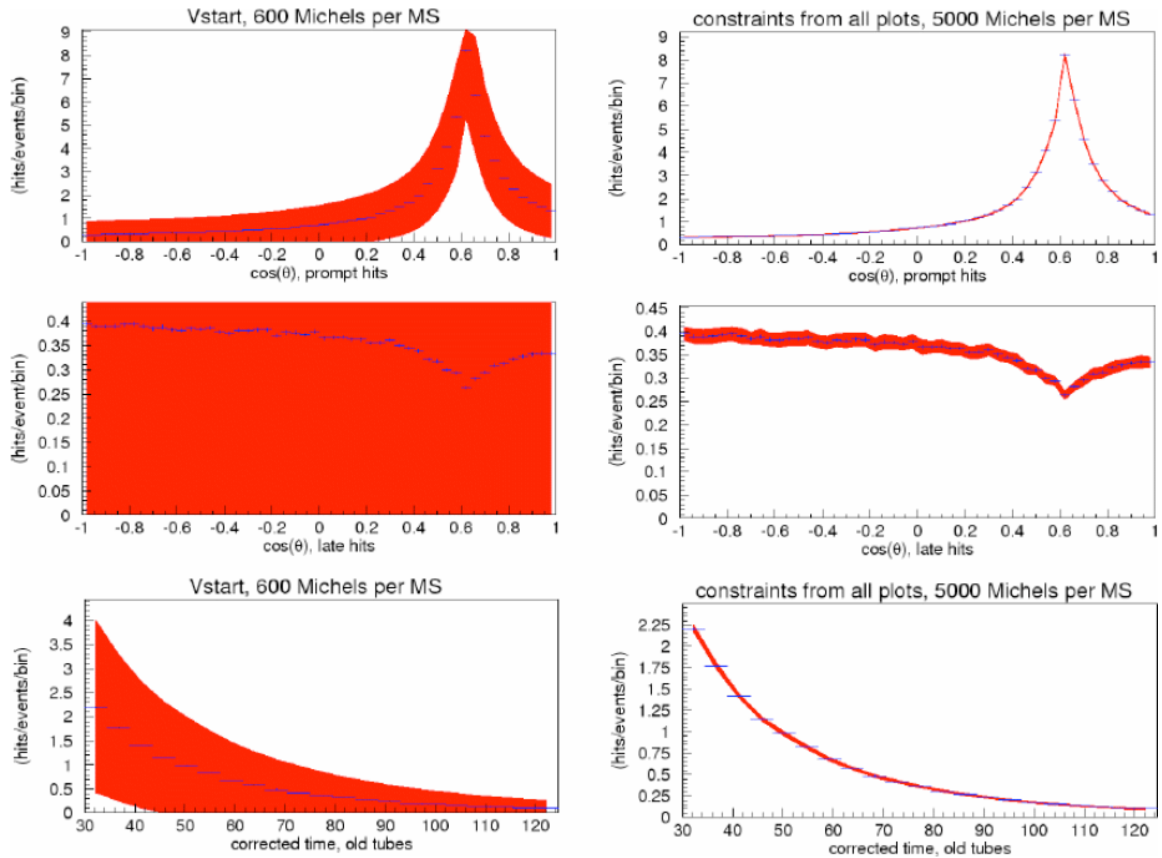
- **finish with μ decay events (low-energy electrons)**
  (~unlimited supply and fast to simulate)

➔ use a Monte Carlo method to reduce uncertainty:

➔ compare data/MC events in relevant distributions for many allowed models

➔ de-weight disallowed regions of model space

➔ NC elastic events help out with scintillation



starting uncertainties in three of the distributions

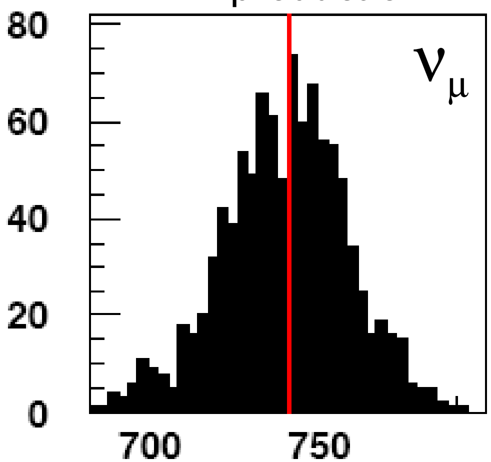
(near) ending uncertainties

"Multisim" approach to assessing systematics

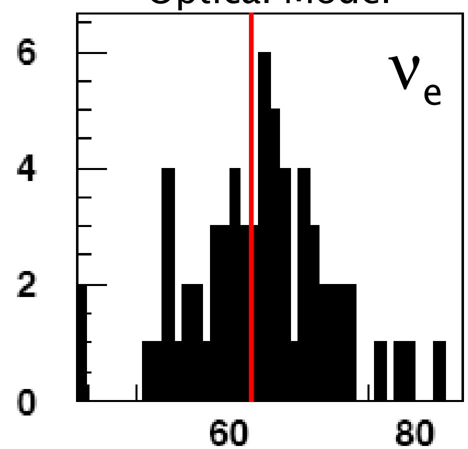
- A multisim is defined as a random draw from the underlying parameter that is considered allowed
- Allowed means the draw does not violate internal or external constraints
- Draws are taken from covariance matrices that dictate how parameters are allowed to change in combination, imagine Cerenkov and scintillation as independent sources of light but requiring the Michel energy to be conserved
- For flux and X-section multisims can be done via reweighting, optical model requires running hit level simulation

1000 multisims for K^+ production

number of multisims



70 multisims Optical Model



red line: standard MC

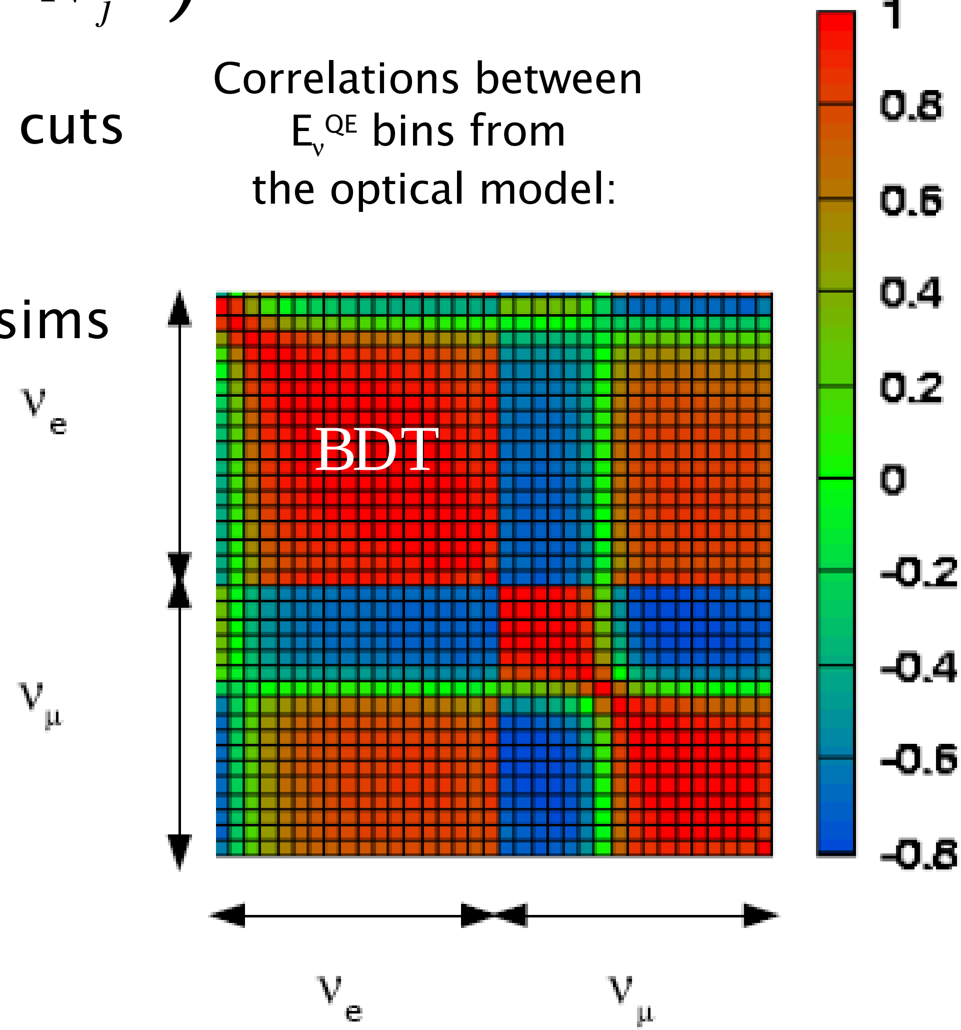
Number of events passing cuts in bin $500 < E_\nu^{QE} < 600$ MeV

Optical model error matrix

$$E_{ij} = \frac{1}{M}\sum_{a=1}^{M}\left(N_i^{a} - N_i^{CV}\right)\left(N_j^{MC} - N_j^{CV}\right)^{MC}$$

- N is number of events passing cuts
- MC is standard monte carlo
- α represents a given multisim
- M is the total number of multisims
- i,j are E_ν^{QE} bins

Correlations between E_ν^{QE} bins from the optical model:



Total error matrix is calculated from the sum of 9 independent sources

TB: ν_e–only total error matrix

BDT: ν_μ–ν_e total error matrix